XAIA INVESTMENT

THE JOINT MODELING OF DEBT AND EQUITY: AN INTRODUCTION

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Abstract In order to detect mispricings between equity and debt instruments referring to the same company, a mathematical model to jointly evaluate both bonds and stock derivatives is required. These so-called credit-equity models are typically not taught at universities in standard lectures on financial mathematics. The present article provides some insights into the mechanics of such models and highlights arising mathematical difficulties.

1 Introduction

The standard market model for the valuation of stock derivatives is still the Black-Scholes model, see Black, Scholes (1973), which turned the field of financial mathematics inside out and founded a whole new branch of mathematical research. Many advancements of the Black-Scholes model question the underlying normality assumption of stock returns, which empirically often cannot be justified. Prominent examples are so-called Lévy-driven models, see Barndorff-Nielsen (1998); Madan et al. (1998); Kou (2002), models with stochastic volatility, see Heston (1993); Barndorff-Nielsen, Shephard (2001), or even combinations of both. Mathematically speaking, the fundamental idea of all these approaches is to model the evolution of the stock exogenously by some more or less complicated stochastic process. Based on this model, prices for stock derivatives such as put and call options are derived by arguments from arbitrage pricing theory.

On the contrary, the valuation of a bond is based on the discounting of its underlying cash flows – a technique established decades before the publication of the seminal work by Black and Scholes. Additional to the usual interest rate discounting, all cash flows due by the bond issuer have to be discounted with the issuer's survival probabilities. In mathematical terms, this means that the future time point of bankruptcy has to be modeled stochastically, and the required survival probabilities must be derived from this model. The in many ways easiest, but also arguable, assumption for this default time is to assume an exponential distribution, i.e. a constant default intensity.

How is it possible to juggle both the valuation of bonds and stock derivatives referring to the same company? To this end, required is a joint mathematical model for both the stock price process and the issuer's default time, a so-called credit-equity model. It is not surprising that such a model can become quite complex, in particular if we postulate that the movements of the stock price have an influence on the survival probabilities of the issuer, and vice versa. For instance, it is intuitive to assume that upon bankruptcy of the issuer, the stock price drops massively, maybe even to zero. Ignoring such fundamental interrelations is clearly not an option, because they form the very basis for every capital structure arbitrage strategy.

The present article surveys some credit-equity models (without claiming completeness), and discusses their properties. Like always in applied mathematics, such models face a natural trade-off between realism on the one hand and tractability on the other hand. Before specific models are presented, Section 2 discusses fundamental requirements the model has to satisfy in order to be both sound and viable. Section 3 discusses the simplest credit-equity model, in which the default time is exponentially distributed. Section 4 is dedicated to so-called $1\,\frac{1}{2}$-factor models, which relax the exponential distribution of the default time by assuming the default intensity to be a function of the stock price. Finally, Section 5 provides a small literature overview of further modeling approaches and Section 6 concludes.

2 Requirements for a credit-equity model

On the one hand, a credit-equity model has to be simple enough in order to allow for efficient pricing of all considered instruments. Otherwise it is of no practical interest at all. On the other hand, it should exhibit as many desirable theoretical properties as possible. This natural conflict of goals is illustrated further in the following by collecting desirable properties.

Desirable theoretical properties:

(a) Upon default of the issuer, the stock price can typically not differ from zero due to technical reasons. When default is imminent but not yet official, the stock price drops significantly.

(b) If the stock price falls significantly, the probability for an imminent default of the issuer should increase. Conversely, this probability should fall with a rising stock price. The basic principle underlying this idea is that the stock price is a good indicator for the economic strength of the company. Of course, this assumption is not always satisfied. For instance, one might think of a leveraged buyout szenario, when the increased demand for debt capital leads to rising default probabilities, but at the same time the expectations of stock holders, and therefore the stock price, might exhibit positive tendencies. Another prominent example is the simultaneous rise of credit default swap spreads and stock price of General Motors in May 2005, see Zuckerman (2005). In “normal“ szenarios, however, a reciprocal relation between default probabilities and stock price is reasonable and desirable.

(c) The probability distribution of stock returns should exhibit many of the stylized properties that have been explored in numerous empirical studies, such as heavy-tailedness and skewness.

Necessary properties with regards to practical viability:

(a) The model is primarily designed in order to evaluate corporate bonds and stock derivatives at the same time. For both evaluations there must be efficient pricing algorithms. One of the adversities is that standard put options, but also bonds in the high-yield sector, and convertible bonds in general, involve American-style exercise features. This means that the maturity of the respective instrument is random and can be dictated by the holder (resp. the issuer). For instance, this maturity depends on the future evolution of the stock price. Financial products with American-style features are typically evaluated with numerical procedures which are based on an approximation of the stock price model by means of a simpler, discrete model. For instance, the Brownian motion, which appears in the Black-Scholes model, can be approximated by so-called random walks, which are motivated by iterated coin tosses. This approximation is known as Donsker's theorem and leads to an efficient binomial tree algorithm for the pricing of American put options in the Black-Scholes setup, see Cox et al. (1979). Interested readers find a mathematically rigorous treatment of this pricing technique in Müller (2009).

(b) Some credit-equity models are still simple enough to allow for the derivation of closed formulas for default probabilities and/or European stock derivatives. Even though this might seem superfluous given there exist accurate numerical pricing algorithms, this is desirable for the following reasons. Firstly, closed formulas for default probabilities typically allow for an evaluation of simple bonds (we mean bonds without American-style features) and credit default swap spreads in fractions of a second. Especially for the model's calibration to market data this computation speed is important. Secondly, the closed formulas can be used as a cross-check for the validity of the numerical approximations required for the pricing of American-style products. This is extremely advantageous, since it facilitates the implementation of the approximative methods. In other words, it is much easier to obtain a feeling for the model's mechanics. Thirdly, closed formulas are often the basis for an exploration of theoretical model properties. One example is the probability that the issuer never defaults. One might be interested in whether this probability can differ from zero at all. Also, statistical properties of the stock returns are much easier to explore when their probability distribution can be computed explicitly, which is sometimes the case.

3 The simplest jump-to-default model

The simplest credit-equity model is the one which is, e.g., proposed in Tsiveriotis, Fernandes (1998) and applied in Kwok, Lau (2004), and is sometimes considered as a market standard, especially for the evaluation of convertible bonds. Unfortunately, this approach completely decouples the equity component from the credit component, which results in quite undesirable economic effects, e.g. because the stock price is not affected at all by a default event. Therefore, in our opinion the simplest reasonable credit-equity model is based upon the Tsiveriotis-Fernandes model, but enhanced by a jump to zero of the stock price upon default, a so-called jump-to-default model. The issuer's default time τ follows an exponential distribution. This means that the survival probability until the (future) time point $t > 0$ is given by $\mathbb{P}(\tau > t) = e^{-\lambda\, t}$, for a model parameter $\lambda > 0$ which is called default intensity[1]. The exponential distribution is a convenient choice in several ways. For example, denoting the continuously compounded zero rate for the time span from today until time $t > 0$ by $R(t)$, the interest rate discount factor for a cash flow at time t is given by $e^{-R(t)\, t}$. In total, i.e. with interest rate discounting and survival discounting, a cash flow due by the issuer at time t has to be discounted by $e^{-(R(t)+\lambda)\, t}$. This means that the default intensity λ can be interpreted as a credit spread on top of the risk-free interest rate term structure $t \mapsto R(t)$. Moreover, the exponential distribution has the nice property that the 2-year survival probability equals the square of the 1-year survival probability. Consequently, the company is neither more nor less likely to default, if it survives the first year. Hence, the exponential distribution can be regarded "piecewise". However, there are a couple of critical aspects with regards to the exponential distribution. Firstly, it might be intuitive to assume that a company is very likely to default in the near future, but that its default probability falls significantly when it survives until a certain time point. Such an effect deviates from the exponential distribution, but can be incorporated by replacing the constant default intensity λ by a (deterministic) function in time $\lambda(t)$. Secondly, the assumption of a constant (or, more generally, deterministic) default intensity implies that the model price of a plain vanilla corporate bond over time is a smooth function, which interpolates between today's value and the value at maturity[2]. In contrast, we observe volatile price evolutions in the market, which are affected by daily fluctuations. Thirdly, a downward trend of the stock price does not translate into an increase in the default intensity, which is always constant (resp. deterministic).

The exponential distribution has nice mathematical properties, but it is often too unrealistic.

This leads us to the modeling of the stock price. The idea of the simple jump-to-default model is to define the stock price process similar to the one of the classical Black-Scholes model before default, when it drops to zero and remains there. However, this intuitive idea is not directly applicable, because the stock price's bear movement to zero destroys the necessary martingale property of the Black-Scholes model. In order to explain this, recall from classical finance theory that the interest rate discounted stock price has to be a so-called martingale under the risk-neutral probability measure, which is relevant for pricing. Loosely speaking, the random stock price process must be risk-adjusted in such a way that on average it yields the same return as a risk-free investment. In the Black-Scholes model, the discounted stock price is defined as $S_t = e^{-0.5\,\sigma^2\, t+\sigma\, W_t}$ with a Brownian motion $\{W_t\}$ and a volatility parameter $\sigma > 0$. This stochastic process $\{S_t\}$ is a martingale, e.g. its expectation $\mathbb{E}[S_t] = S_0$ is identically constant, irrespective of $t > 0$. It is postulated that the default time is independent of the Brownian motion $\{W_t\}$. Assu-

[1] This nomenclature is justified by Formula 3, see below.

[2] In the case of clean pricing. If dirty pricing is considered, the price jumps at coupon payment dates.

ming the process $\{S_t\}$ jumps to zero at time τ and remains there after τ, for each $t > 0$ this leads to a positive probability that $S_t = 0$. In order to generate a constant expectation $\mathbb{E}[S_t] = S_0$ this probability has to be rebalanced, by adjusting the drift of the stock price process accordingly. Consequently, the discounted stock price process is modeled as

$$S_t = S_0 \, e^{(\lambda - \frac{1}{2}\sigma^2)\, t + \sigma W_t} \, 1_{\{\tau > t\}}, \quad t \geq 0.$$

This means that the discounted stock price process before the default time τ satisfies the stochastic differential equation

$$dS_t = S_t \, (\lambda \, dt + \sigma \, dW_t), \quad t < \tau. \tag{1}$$

One can show that $\{S_t\}$ is a martingale, and per definitionem it drops to zero upon default. Before default the discounted stock price process has positive drift λ under the risk-neutral pricing measure.

Positive model properties:

(a) There are closed formulas for plain vanilla stock options, which basically arise from a small adjustment of the Black-Scholes formula. Interestingly, this tiny model already suffices to create implied volatilities which exhibit a strong skew with respect to the exercise price and fit much better to market data than the classical Black-Scholes model, see Figure 1.

(b) The model is simple and intuitive. In particular, the default intensity λ can be considered as credit spread, which a company has to pay on top of the risk-free rate in order to compensate for the riskiness.

(c) For the pricing of American-style stock derivatives it is possible to extend the well-known binomial approximations from the classical Black-Scholes model to the jump-to-default setup. This can be done in exactly the same way as described in, e.g., Cox et al. (1979); Müller (2009), only with respective drift adjustment.

Negative model properties:

(a) As mentioned earlier, there is no feedback from the stock price movement to the default intensity. This means that a downward trend of the stock price is not reflected in increasing default probabilities.

(b) The model-implied bond price evolution is a smooth function over time[3], which is not in accordance with the observable volatile price evolutions in the market. This simply follows from the default intensity λ being constant (resp. $\lambda(t)$ deterministic).

(c) The first bullet point (a) above is not only a theoretical flaw, also for practical applications such as the hedging of a bond with stock options there can be dramatic consequences. For instance, the price of a plain vanilla coupon bond without conversion features does not depend on the stock price at all, i.e.

[3] Cf. Figure 1 in Kwok, Lau (2004).

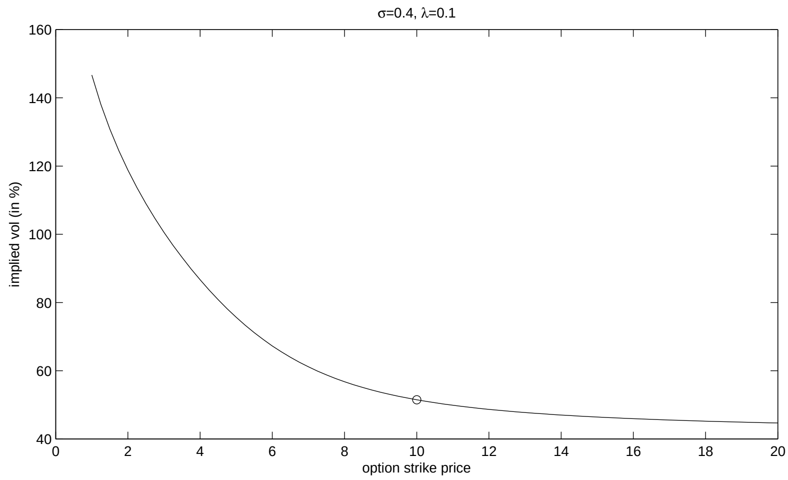

Fig. 1: Implied volatilities for European puts/calls with maturity 1 year in the simple jump-to-default model, where $S_0 = 10$ and $\lambda = 0.1$, $\sigma = 0.4$.

it has a stock price delta of zero! The price of a convertible bond does depend on the stock price, but only via the implicit stock call option, not via the bond component of the convert. To illustrate this further, let us assume we have bought a convertible bond. We assume that a downward trend of the stock price implies a downward trend of the convertible bond price. To avoid such a scenario, our idea is to build a hedge position by buying put options on the stock. To compute the number of put options required in order to balance the price movements of the bond, we must compute the sensitivity of the bond price with respect to stock price movements within our model (the so-called equity-delta of the bond). Figure 2 illustrates this sensitivity for an exemplary convertible bond, in the simple jump-to-default model as well as in a more realistic $1\,\frac{1}{2}$-factor model, see the next section. It is obvious that in the simple jump-to-default model the bond price suffers much less from a strong decline of the stock price than in the $1\,\frac{1}{2}$-factor model. Since in many cases it is highly likely that a significant downward move of the stock triggers a significant downward move of the bond as well, the use of the simple jump-to-default model might lead to a hedging position that is not sufficient to compensate for the losses.

4 Models with $1\,\frac{1}{2}$ factors

The simple jump-to-default model from the previous paragraph is typically referred to as a one-factor model, because over time the only random stochastic process involved is the stock price process. Other risk factors like interest rates or the default intensity are modeled deterministically, i.e. it is assumed that their complete movements over time are known by now already, which in reality is clearly not the case. The reason for this modeling ansatz is simple: models with more than one stochastic driver are much more difficult to implement. The so-called $1\,\frac{1}{2}$-factor models offer an elegant solution for a stochastic modeling of the default intensity without introducing a second stochastic driver. It is assumed

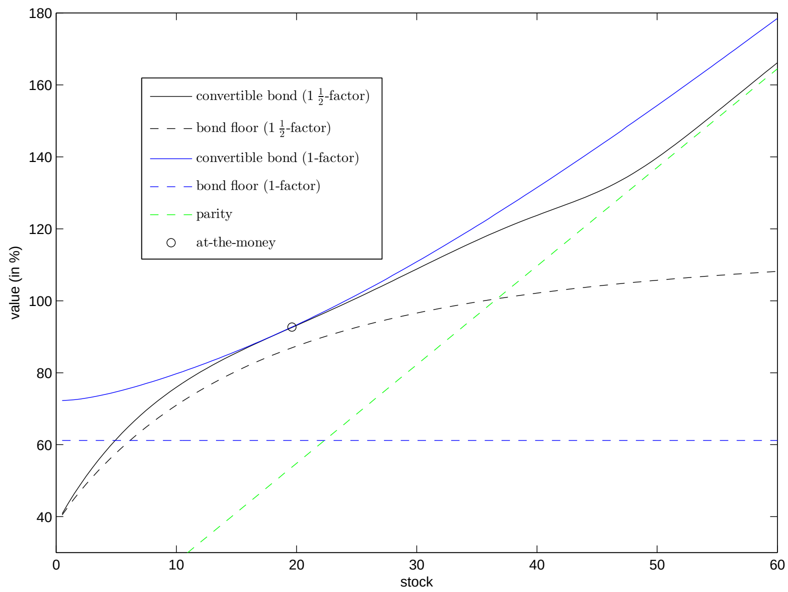

Fig. 2: Sensitivity of an exemplary convertible bond, computed with the simple jump-to-default model as well as with the $1\,\frac{1}{2}$-factor model by Carr, Linetsky (2006). The parameters in both models are chosen such that the current market value of the convertible bond is matched. The dotted lines illustrate the sensitivity of the so-called bond floor, i.e. the same bond without conversion right. One observes that the latter does not depend on the stock price in the simple jump-to-default model.

that the default intensity λ at time t is given by a function h of the stock price S_t, so to speak a "half factor". A common choice for the functional relation h between stock price and default intensity is

$$\lambda(t) = h(S_t) = \lambda_0 \left(\frac{S_t}{S_0}\right)^{-p}, \quad t \geq 0, \tag{2}$$

The idea of $1\,\frac{1}{2}$-factor models is to model a reciprocal relationship between stock price and default intensity.

for model parameters $\lambda_0 > 0$ and $p > 0$. As desired, this implies that with increasing (decreasing) stock price the default intensity decreases (increases). However, a transformation of this intuitive idea into a rigorous mathematical model is significantly more demanding than in the simple jump-to-default case of the previous paragraph. What precisely is a stochastic default intensity, and how is a default time with such a default intensity defined mathematically? The answers to these questions are explained in the sequel. For all $1\,\frac{1}{2}$-factor models considered in the present article we consider a probability space supporting a Brownian motion $\{W_t\}$ and an independent exponential random variable ϵ with mean 1. However, in contrast to the previous paragraph, the exponential random variable ϵ is not directly interpreted as default time, but only used as building block. The issuer's default time τ is defined as a function of both objects, $\{W_t\}$ and ϵ, by means of the following canonical construction[4]:

$$\tau := \inf\Big\{t > 0 \,:\, \int_0^t \lambda(s)\, ds > \epsilon\Big\}. \tag{3}$$

In words, the default time is defined as the first time point at which the integrated default intensity exceeds the random trigger level ϵ. The essential idea of this mathematical construction is explained by the following approximation formula, which is valid for small $\Delta > 0$:

$$\mathbb{P}(\tau \leq t + \Delta \,|\, \tau > t) \approx \lambda(t)\, \Delta. \tag{4}$$

The default intensity can be thought of as an instantaneous default likelihood.

Intuitively, this formula says that the instantaneous default probability (i.e. the default probability within the next second) is proportional to the default intensity, which after all justifies its nomenclature. For instance, if the parameterization (2) is opted for, a decline of the stock price implies an increase of the instantaneous default probability, and vice versa.
As a direct extension of (1), the discounted stock price process is defined under the risk-neutral pricing measure as the solution to the stochastic differential equation

$$dS_t = S_t\,(h(S_t)\, dt + \sigma\, dW_t), \quad t < \tau. \tag{5}$$

For time points $t \geq \tau$ after default it is assumed that $S_t = 0$. Again, the default intensity $\lambda(t) = h(S_t)$ plays the role of a drift adjustment in order to guarantee the martingale property of the stock price process. The difference now is that this drift depends itself on the stock price process, i.e. is random. If the function h is chosen identically constant, this results as a special case in the simple jump-to-default model of the previous paragraph. In this sense, $1\,\frac{1}{2}$-factor models can be considered proper extensions of the simple jump-to-default model. Apparently, all aforementioned negative model properties of the simple jump-to-default model are eliminated when replacing it by a $1\,\frac{1}{2}$-factor model:

(a) The stock price movements directly influence the default intensity.

(b) The price evolution of a bond is a function of the default intensity, which inherits its volatile characteristics from the Brownian motion of the stock price diffusion.

(c) If the parametric form (2) is chosen, the default intensity becomes infinitely large when the stock price tends to zero. Consequently, the bond price drops dramatically, namely it converges to the value which is expected to be received from the remaining assets of the insolvent corporation. This effect is illustrated in Figure 2, where a recovery assumption of 40% of the invested nominal is assumed.

It is important to mention that these desirable properties do not come for free but create a battery of mathematical difficulties. Coping with them is a necessary requirement in order to implement the respective credit-equity model. The biggest challenges, whose solutions sometimes are based on deep probability theory, are listed in the sequel:

A $1\,\frac{1}{2}$-factor model is mathematically much more demanding than the simplest jump-to-default model.

[4] The default intensity $\lambda(t)$ has to satisfy certain conditions.

(a) Stock returns (before default) are no longer normally distributed because of the random drift component. Their probability distribution can be computed explicitly in some cases, but even then is challenging to handle numerically.

(b) Closed formulas for European stock options and default probabilities are difficult to obtain. In some cases explicit formulas are known, but numerical evaluations on the PC are not always straightforward due to the appearance of special functions such as hypergeometric functions. In common specifications one can show for the default time τ, defined in (3), that

$$\mathbb{P}(\tau > t) = \mathbb{E}\Big[e^{-\int_0^t h(S_u)\,du}\Big],$$

which indicates why closed formulas for survival probabilities are difficult to obtain.

(c) Unlike in the classical Black-Scholes model, because of the random drift term $h(S_t)$ the diffusion process describing the stock price before default might become zero. This is unreasonable and must therefore be avoided. In some model specifications one can show mathematically that even if the diffusion process might hit zero, the default time, defined via (3), almost surely occurs earlier. This resolves the problem, since the stock price is defined to be zero after default anyway. However, such a mathematical proof is far from trivial.

(d) For the evaluation of American-style instruments discrete approximations of the stock price process are required, which are not as easy to extract from Donsker's theorem as in the simple jump-to-default model. For quite general diffusion processes there exist good approximation algorithms, however, rigorous treatments of their accuracy and efficiency are sparse. Similarly, numerical approaches based on partial differential equations are often applied heuristically, i.e. without rigorous mathematical investigation.

References on $1\,\frac{1}{2}$-factor models.

For interested readers we'd like to recommend a couple of references, which work with $1\,\frac{1}{2}$-factor models. There is a series of papers by Bielecki, Crépey, Jeanblanc and Rutkowski which treat the subject with great generality in a very rigorous manner, see Bielecki et al. (2008a,b, 2009, 2011). The cost for the great generality in these references is a massive mathematical apparatus which is tedious to digest for the practically oriented reader. For the model defined via (2) and (5) closed formulas for default probabilities and European stock derivatives are derived in Linetsky (2006), who introduces the model in a very precise and rigorous manner. The same model is also applied by the earlier references Andersen, Buffum (2004); Ayache et al. (2003) for the valuation of convertible bonds. These references are much more vague and practically oriented, and propose an ansatz via partial differential equations. Also worth mentioning is the model by Carr, Linetsky (2006), who incorporate an additional so-called CEV-feature (constant elasticity of variance) into the stock price process. Intuitively, this feature implies that the volatility of the stock price process stands in reciprocal relationship with its level. Although such an additional desirable property is expected to cause further difficulties, the opposite is the case: closed formulas for European stock options as well as the numerical implementation of pricing algorithms for American-style features become easier. The reference Carr, Madan (2010) goes even one step further by allowing for arbitrary local volatility specifications in the stock price process.

5 Further models

We'd like to use the opportunity to draw the readers' attention to a model which defines the default time differently, but from a theoretical perspective is at least equally valuable as $1\,\frac{1}{2}$-factor models. It is introduced in Chen, Kou (2009), who define the firm's asset value process as a jump diffusion, and the default time as the first time point at which this process drops below a certain barrier, namely the aggregated amount of the company's liabilities. The stock price process itself has to be derived in this model as the difference between the firm's market value minus its total debt, which makes it less tractable for the pricing of stock derivatives. Such a model is called a structural model, since the default time is defined explicitly as a function of real-world quantities (the firm's assets and liabilities). The origin of this model class is formed by the seminal references Merton (1974); Black, Cox (1976), which have been developed further significantly since then. In contrast to structural models, the $1\,\frac{1}{2}$-factor models discussed in Section 4 are referred to as reduced form models, since the default time is defined implicitly by means of a purely theoretical trigger variable ϵ, see[5] (3). On the one hand, Chen, Kou (2009) show that the induced default probabilities have similar properties as the ones in the $1\,\frac{1}{2}$-factor model (2) and compute several relevant quantities in closed form. On the other hand, this reference answers questions regarding the optimal debt-equity ratio for the company's balance sheet, which after all makes it an article worth reading.

The $1\,\frac{1}{2}$-factor models have to be distinguished from the class of structural, Merton-type models.

Finally, there are some more involved modeling approaches that are based on more stochastic factors, e.g. stochastic interest rates, stochastic volatility or "really" stochastic default intensity. Such models are typically more realistic but also more complicated to work with. Davis, Lischka (2002) consider a $2\,\frac{1}{2}$-factor model in which, in generalization of the $1\,\frac{1}{2}$-factor models, stochastic interest rates are considered. Further mentionable references in this direction are Carr, Wu (2009), who consider a 3-factor model for stock price, stock volatility and default intensity, and Kovalov, Linetsky (2008), who even go one step further and additionally include a stochastic interest rate model as fourth factor.

[5] Also, reduced form models do not consider the relation between default and firm value in an explicit manner.

6 Conclusion

We provided a survey of credit-equity models, which are applied in order to jointly evaluate stock derivatives and bonds of the same company. Special focus was devoted to 1- and $1\,\frac{1}{2}$-factor models, and the readers were hinted at mathematical challenges that have to be faced. Finally, we provided a list of references (surely incomplete).

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