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The upfront-adjusted par spread for credit default swaps

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Date: May 27, 2022

Abstract The par spread for a credit default swap (CDS) is an annualized measurement for the cost of protection against a credit event with respect to the underlying reference entity. According to the so-called "credit triangle", it approximately equals the product of default likelihood and loss given default in notional terms. However, due to the market convention of standardized coupons, the loss potential tends to zero as the default likelihood tends to infinity, since the upfront compensation at settlement becomes huge and reduces the downside risk for the protection seller. Unfortunately, the par spread increases to infinity in this case and thus fails to reflect the upfront payment market convention appropriately. The present article introduces the *upfront-adjusted par spread*, which overcomes this problem.

1 Motivation

We assume the reader is familiar with a credit default swap (CDS), otherwise we refer to the introductory textbook Felsenheimer et al. (2019). CDS prices are usually communicated in terms of the so-called par spread between traders, which is formally introduced below in Section 2. Intuitively, the par spread is an annualized measurement for the cost of CDS protection, which means that per unit of contract notional one pays the par spread every year until maturity, in case no credit event takes place. However, the actual annualized insurance premium is standardized, usually to either 1% or 5% of the notional, and the protection seller receives an upfront payment at settlement in order to account for the difference between par spread and standardized coupon rate. Consequently, this upfront payment is the fair market price of the CDS. On first glimpse, the par spread is a simple and intuitive quantity that facilitates a quick understanding of the CDS in concern. On second glimpse, however, there are certain shortcomings of the par spread concept that we seek to point out and overcome in the present article. These are:

(i) **Over-estimation of risk in distressed situations:** When a soon default becomes very likely, the par spread explodes to infinity. This suggests that the cost of CDS protection can become infinitely large. Since the protection seller receives the large CDS price immediately at settlement, however, the loss risk decreases dramatically. This risk-reduction effect is completely ignored by the par spread.

(ii) **Problem with regards to comparison of CDS portfolios:** Consider a CDS portfolio manager that seeks to benchmark her investment with an equally-weighted CDS portfolio, such as one based on a common CDS index. A fair performance comparison of the two portfolios presupposes that they are "equally risky" in some meaningful sense. An obvious idea is to compare portfolios with the same average par spread. But the aforementioned over-estimation of risk becomes even more problematic in the portfolio context. If just one portfolio constituent becomes highly distressed, its exploding par spread (if well-defined at all) leads to an explosion of the average par spread as well, even though the potential loss in case of this constituent's default is limited to the constituent's portfolio weight in notional terms.

2 Notation, definition, and intuition

We denote by u the upfront price of the CDS and by c the running premium, which is usually standardized to 1% or 5%, both u and c quoted in percent of the CDS notional. The maturity of the CDS in years is denoted by T. In order to keep everything as simple as possible, we assume a flat interest rate r for discounting, continuous coupon payments, and a fixed recovery rate assumption R in percent of the CDS notional. Furthermore, like in the standard ISDA model we introduce a parameter $\lambda > 0$ that is uniquely determined by the equation

$$u = \int_0^T (1-R)\,\lambda\, e^{-(\lambda+r)\,t}\,\mathrm{d}t - \int_0^T c\, e^{-(\lambda+r)\,t}\,\mathrm{d}t. \qquad (1)$$

Intuitively, the right-hand side equals the difference between expected net present value of default compensation payment and expected net present value of coupon payments to be made, under the assumption that the future random time point of a credit event has an exponential distribution with rate λ. Consequently, Formula (1) reflects the so-called equivalence principle from insurance mathematics, which specifies the (upfront) price u as expected difference between compensation and premium payments. We may view the right-hand side in Formula (1) as a function $f_R(\lambda)$ of the parameter λ and simplify to

$$f_R(\lambda) = \begin{cases} \frac{(1-R)\,\lambda - c}{\lambda + r}\Big(1 - e^{-(\lambda+r)\,T}\Big) & \text{, if } \lambda \neq -r \\ \big((1-R)\,\lambda - c\big)\,T & \text{, else} \end{cases}.$$

We write f_R in order to highlight the dependence of f_R on the assumed recovery rate R. It is not difficult to see that $f_R(\lambda)$ is strictly monotonically increasing in λ with $\lim_{\lambda\to\infty} f_R(\lambda) = 1-R$. Consequently, Equation (1), which in short is $u = f_R(\lambda)$, always has a unique solution, provided $u \leq 1-R$, which we henceforth assume. The following definition introduces the main object of study in the present article.

Definition 2.1 (Par spread and upfront-adjusted par spread) For given market upfront price u and recovery rate R we denote by $\lambda = f_R^{-1}(u)$ the unique number satisfying $u = f_R(\lambda)$.

(a) Let R be the recovery rate assumption in the standard ISDA model, which is a function of the CDS standard and the CDS seniority, typically $R \in \{20\%, 25\%, 40\%\}$. The *par spread* is defined to be $s = \lambda\,(1-R)$.

(b) Let R be the market's recovery rate assumption. The *upfront-adjusted par spread* is defined to be $\tilde{s} = \lambda\,(1-R-u)$.

The par spread is so common in the marketplace that in most cases the CDS price u is not even communicated but instead traders agree upon a par spread. However, it is not well-defined as soon as the market price u exceeds the value $1 - R$ with R from the standard ISDA model. This is a typical situation for highly distressed names, and market participants in this situation switch to price communication in terms of u. The upfront-adjusted par spread is our definition, which we introduce with the intention of overcoming some weaknesses of the par spread that were already mentioned in the abstract and introduction.

Remark 2.2 (Intuition)
If a CDS was traded like an interest rate swap at zero initial market value, the par spread was precisely the running coupon rate to replace the standardized rate c (under the exponential distribution assumption implicit in the ISDA model). In fact, this is the historical origin of the par spread methodology, as CDSs were traded this way before 2003. According to this intuition, s equals precisely the premium the protection buyer has to pay per annum on the notional. However, the market convention of standardized coupon rate c and upfront compensation at settlement changes the risk profiles of protection seller and buyer. This is particularly important for highly distressed names, where the upfront payment reduces the risk of the protection seller dramatically. This risk reduction is not reflected appropriately in the par spread. In contrast, the idea of the upfront-adjusted par spread is to account for this risk reduction. The parameter λ is a rough estimate for the one-year default probability according to the first-order Taylor approximation $\lambda \approx 1 - \exp(-\lambda)$. The idea for the definition of s according to the so-called "credit triangle" is to define the par spread as product of the default likelihood proxy λ and the loss given default. But the value $1 - R$ in the definition of the par spread s only equals the loss given default in notional terms, but does not account for the risk reduction due to the fact that the upfront is immediately consumed by the protection seller. In contrast, the upfront-adjusted par spread $\tilde{s}$ does precisely that by replacing $1 - R$ with the value $1 - R - u$, which is the actual amount of the nominal that is at stake for the protection seller. Especially for large u (i.e. highly distressed names) this risk reduction is significant.

The following lemma collects the main properties of the upfront-adjusted par spread $\tilde{s}$ when viewed as a function of the market price u.

Lemma 2.3 (Anatomy of $\tilde{s} = \tilde{s}(u)$)
If we consider the upfront-adjusted par spread $\tilde{s} = \tilde{s}(u)$ and the par spread $s = s(u)$ as functions of the market price u, we have $\tilde{s}(0) = s(0) = c$ and $\lim_{u \to 1-R} \tilde{s}(u) = c + r\,(1 - R)$. Furthermore, there is a critical value $u_* < 1 - R$ at which $s(u_*)$ is maximal, and $s(u)$ increases (decreases) before (after) u_*.

Proof
Since $f_R(\lambda)$ is a monotonically increasing function in λ, we may as well study the function

$$\tilde{s}\big(f_R(\lambda)\big) = \lambda\,\big(1 - R - f_R(\lambda)\big)$$

as a function of λ. We have that $u \to 1 - R$ is equivalent to $\lambda \to \infty$, and we observe for $\lambda > -r$ that

$$\tilde{s}\big(f_R(\lambda)\big) = \frac{\lambda\,(1-R)}{\lambda + r}\left\{\lambda\, e^{-(\lambda+r)\,T} + r + \frac{c}{1-R}\left(1 - e^{-(\lambda+r)\,T}\right)\right\},$$

an expression that is easily seen to converge to $c + r\,(1-R)$ for $\lambda \to \infty$, as claimed. The claim on the unique critical point u_* remains to be proven, but can be verified from Figure 1. □

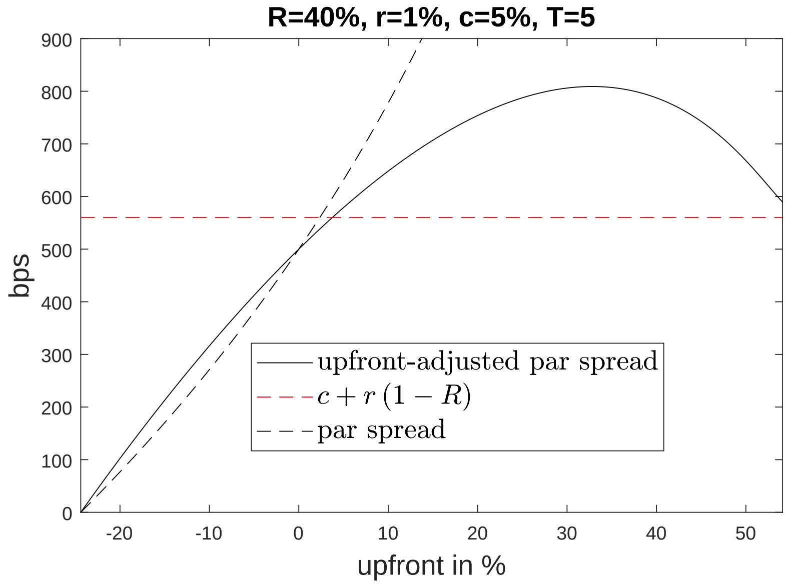

Fig. 1: Depiction of the upfront-adjusted par spread $\tilde{s}$ and the par spread s as a function of the upfront price u.

References

J. Felsenheimer, W. Klopfer, U. von Altenstadt, Credit default swaps [in German only], Wiley (2019).