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THE RISK REVERSAL TRADE ON INSOLVENT FIRMS

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Abstract After a heavily indebted company has filed for bankruptcy, its stock price typically drops significantly and enters a new regime. On the one hand, the stock price is expected to approach the value zero eventually, since the remaining firm value is ultimately distributed among its creditors with minimal leftovers for equity owners. On the other hand, until a negligible stock price level is attained, speculative trading can lead to high volatility and even extreme upward spikes. Recent examples for such situations are the stock prices of Hertz Global Holdings Inc. and Wirecard AG. A bullish risk reversal trade is one possible alternative to an outright stock purchase that speculates on such a spike. Similarly, a bearish risk reversal trade is one possible implementation to technically short-sell the stock. In both cases, the essential profile implemented by the risk reversal trade mimics that of an equity forward, with the sole distinction that typical put and call options in the marketplace are American-style. The essential risk is whether the involved options expire before or after the stock price has arrived at an absorbing state of negligible value. In order to assess the risk reversal's attractiveness traders thus need to compare their own subjective opinions regarding the equity forward with those implied by the observed put and call prices. The task of backing out the equity forward from American-style prices is a non-trivial numerical challenge in general. The present article introduces a method to compute model-free lower and upper bounds on the equity forward that are sharp, i.e. attained by some arbitrage-consistent models.

1 Organization of the article

Section 2 introduces general background on American-style put and call options, as well as on the put-call parity. It furthermore provides model-free bounds for the equity forward implied from American-style put and call options in a concrete practical example, namely Wirecard AG. The theoretical derivation of these bounds is then presented in Section 3, which is more oriented towards technically interested readers. Finally, in Section 4 we present a list of open research questions that are closely related to the presented material.

2 Background and motivation

The classical put-call parity for European-style vanilla options allows market participants to back out the market's expectation about the future stock price level, the so-called *equity forward*, from observed option prices. Formally, if (S_t) denotes the stock price process with current stock price S_0, the equity forward with maturity T is defined to be $F(0,T) := \mathbb{E}[S_T]$. In this definition, the expectation operator is taken with respect to a risk-neutral probability measure, meaning that under this measure the observed European-style put and call options are given by the expected values of their discounted payoffs. This classical definition of equity forward is model-free in the sense that it is invariant with respect to the involved risk-neutral measure. If P and C denote the prices of European-style put and call options, both with identical maturity T and strike K, the put-call parity reads

$$F(0,T) = K - e^{r\,T}\,(P - C), \tag{1}$$

where $r \in \mathbb{R}$ is an interest rate used for discounting future cash flows at time T. Consequently, $F(0,T)$ can be read off from observed put and call prices without the need for a stochastic model for (S_t).

Unfortunately, in practice observed put and call prices are almost always American-style. Due to possibly non-negligible values of the involved early exercise premia the put-call parity does no longer hold. Instead, the equality in (1) can only be replaced by an inequality. In this regard, the following facts are well-known:

- Upper bound on forward:
  - $r \geq 0$: Guo, Su (2006) and also Desmettre et al. (2017) show that $F(0,T) \leq e^{r\,T}\big(K - (P - C)\big)$.
  - $r \leq 0$: The American put is known to be not exercised early, hence trivially $F(0,T) \leq K - e^{r\,T}\,(P - C)$ from (1), since the price of the American-style call option is greater or equal than that of the European-style call option.
  - Summarizing both cases, we obtain the well-known general model-free upper bound

    $$F(0,T) \leq \max\left\{1, e^{r\,T}\right\} K - e^{r\,T}\,(P - C).$$

- Lower bound on forward:
  - Without dividends: the American call is known not to be exercised early, hence trivially $F(0,T) \geq K - e^{r\,T}\,(P - C)$ from (1), since the price of the American-style put option is greater or equal than that of the European-style put option.
  - With dividends[1]: we are not aware of a model-free bound. This is presumably due to the fact that the European-style put-call parity (1) a priori only helps to estimate $F(0,T)$ from above, making use of the fact that the American-style call price is larger than the European-style price.

An equity forward contract might not always be available in the marketplace. One possibility to implement a short view on the stock price in such a situation is a bearish risk reversal: buy the

[1] According to our motivation with insolvent firms, one might believe that the absence of dividends is a natural assumption. However, in such highly distressed situations one typically observes very significant sell-interest and hence repo rates, that technically enter the pricing models in the same form as dividends do.

put option and sell the call option, with identical strike and maturity. Unlike in the case of European-style options, however, this trade's payoff at time T does not necessarily replicate the payoff $K - S_T$ of a European-style bearish risk reversal, since one faces the risk to get called early. At such potential time of early exercise one has to buy the stock to deliver it into the call and sell a new (American-style) call option with same strike and maturity, in order to keep the trade alive until maturity T. The quantification for such risk relies on a stochastic model for the stock price process and associated numerical routines to compute American-style option prices. Seminal references dealing with American-style option pricing include Jacka (1991) (within the Black-Scholes model) as well as Carr et al. (1992); Detemple, Tian (2002); Broadie, Detemple (2004) in more general models. A trader has a subjective opinion about the expected value of S_T, and a reasonable quantitative criterion for entering into the bearish risk reversal is an option-implied equity forward that is significantly above her subjective opinion.

Whereas a bullish risk reversal trade is always an alternative to buying the stock that requires less liquidity, a bearish risk reversal might sometimes be the only technical possibility to short-sell the stock, since equity forwards are not traded. A typical situation, in which this is the case, occurs for heavily indebted companies who have recently filed for insolvency. To provide a concrete example, on 30 June 2020 a bearish risk reversal trade on Wirecard AG, who has filed for insolvency one week before, could be exercised at the following levels: option maturity $T = 0.4685$ (18 Dec 2020), discounting rate $r = -0.003$ (approximately 6m Euribor), option strike price $K = 5$, reference stock price $S_0 = 5.1701$, American-style put option price $P = 3.78$, American-style call option price $C = 2.2609$.

With the argument presented in the main body of the article we can prove that the equity forward $F(0,T) := \mathbb{E}[S_T]$ is necessarily bounded by

$$1.2253 \leq F(0,T) \leq 3.4832,$$

with the sole assumption being made that the observed prices are free of arbitrage. Furthermore, both bounds are sharp, i.e. we can construct arbitrage-consistent models attaining the bounds. In words, under the assumption that the observed prices for put and call option are free of arbitrage the market's expected value of the stock price on 18 Dec 2020 is necessarily in the interval $[1.2253, 3.4832]$. Admittedly, this seems to be a huge interval on first glimpse. However, for the bearish risk reversal trader especially the lower bound of 1.2253 is of interest. Given the insolvency and the expectation that equity owners are left with almost zero value ultimately, the essential risk of the trade is the timing: will the stock value already be negligibly small until T, or does it take longer than that? If the trader believes that by time T the stock price will certainly be lower than 1.2253, then the bearish risk reversal trade is good, independent of any model assumption.

Our argument to bound $F(0,T)$ relies on two simple, "extreme" models that attain the bounds and mimic the worst/best situations for the risk reversal trader. It furthermore relies on negative interest rate $r \leq 0$, since this implies that European-style put options have the same value as their American-style counterparts. Luckily, this situation $r \leq 0$ is currently the more interesting one, at least in the Eurozone.

3 Model-free bounds for $F(0,T)$ and their sharpness

We assume a flat and known interest rate[2] $r \leq 0$ for discounting throughout, and we consider a stock which continuously pays out an unobserved rate $\delta \geq 0$, through continuous dividend payments and/or via proceeds from stock lending (repo rate). We observe the prices $C_0(K,T)$ and $P_0(K,T)$ of American-style call and put options with strike K and maturity T on the stock with current price S_0. We further assume that the two prices are arbitrage-free. According to arbitrage pricing theory, there exists a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{Q})$ and a stochastic process $(Y_t)_{t\geq 0}$ with the following properties:

(i) The stock price process $(S_t)_{t\geq 0}$ satisfies $S_t = e^{(r-\delta)\,t}\,Y_t$ for $t \geq 0$.

(ii) The stochastic process $(Y_t)_{t\geq 0}$ is a martingale.

(iii) The American-style put and call options satisfy

$$C_0(K,T) = \operatorname*{ess\,sup}_{\tau \in \mathcal{T}[0,T]} \Big\{ e^{-r\,\tau}\,\mathbb{E}\Big[\Big(e^{-(r-\delta)\,t}\,Y_\tau - K\Big)_+\Big]\Big\},$$
$$P_0(K,T) = \operatorname*{ess\,sup}_{\tau \in \mathcal{T}[0,T]} \Big\{ e^{-r\,\tau}\,\mathbb{E}\Big[\Big(K - e^{-(r-\delta)\,t}\,Y_\tau\Big)_+\Big]\Big\},$$

where $\mathcal{T}[0,T]$ denotes the set of all $(\mathcal{F}_t)$-stopping times taking values in $[0,T]$ almost surely.

The rate δ essentially determines the equity forward, which is given by

$$F(0,t) := \mathbb{E}[S_t] = S_0\,e^{(r-\delta)\,t}, \quad t \geq 0. \tag{2}$$

It is important to notice that the pair (δ, Y) needs not be uniquely determined by the observed prices. This means there can be different stock price models $S = (\delta, Y)$ explaining the prices. In fact, the goal of our investigation is to determine a possible range for the parameter δ which is consistent with observed prices.
We have already mentioned before that a model-free upper bound on the forward can be derived from (1), and is given by

$$F(0,T) \leq K - e^{r\,T}\,\big(P_0(K,T) - C_0(K,T)\big).$$

Plugging in (2) and solving for δ yields a model-free lower bound for δ, given by

$$\delta \geq \delta_\ell := -\frac{1}{T}\,\log\Big(\frac{K - e^{r\,T}\,\big(P_0(K,T) - C_0(K,T)\big)}{S_0}\Big) + r.$$

Furthermore, the derivation based on the put-call-parity for European-style options makes clear that this bound is sharp, whenever it is possible to find at least one model $\tilde{S} = (\delta_\ell, \tilde{Y})$ satisfying

[2] For the currently less interesting case $r > 0$ the presented logic needs to be revised, which is subject of further research.

(i), (ii), and (iii) such that both put and call are never exercised, i.e.

$$C_0(K,T) = e^{-r\,T}\,\mathbb{E}[(\tilde{S}_T - K)_+],$$
$$P_0(K,T) = e^{-r\,T}\,\mathbb{E}[(K - \tilde{S}_T)_+].$$

It is an open question whether it is always possible to find such a model, or to state conditions under which this is always possible. However, in concrete applications one may try specific “simple” models and see if they do the job. We derive one such candidate model below in Section 3.2. Before we do this, however, we focus on a lower bound for $F(0,T)$, respectively an upper bound for δ.

3.1 Upper bound on δ

The following result is one of our major findings. It derives a sharp and model-free upper bound on δ given the prices $P_0(K,T)$ and $C_0(K,T)$.

Theorem 3.1 (Model-free upper bound on δ)
Let (δ, Y) be a stock price model satisfying (i), (ii) and (iii). In addition, we assume non-positive interest rate $r \leq 0$. Then necessarily

$$\delta \leq \delta_u := -\frac{1}{T}\,\log\left\{\frac{K\,e^{-r\,T} - P_0(K,T)}{S_0}\right\}$$

and there exists a model $(\delta_u, \tilde{Y})$ satisfying (i), (ii), and (iii).

Reformulating the last theorem in terms of the equity forward, we obtain the model-free and sharp lower bound

$$F(0,T) \geq K - P_0(K,T)\,e^{r\,T}.$$

This lower bound only depends on the put price and is sharp, i.e. cannot be improved without further assumptions or information (beyond $P_0(K,T)$ and $C_0(K,T)$).

Proof
We first prove that the condition

$$K + C_0(K,T) \leq S_0 + P_0(K,T)\,e^{r\,T} \tag{3}$$

generally holds. Assuming that it does not hold, we derive an arbitrage. To this end, assume

$$K + C_0(K,T) > S_0 + P_0(K,T)\,e^{r\,T}$$

and an arbitrageur buys $e^{r\,T}$ put options and one stock, while selling one call and a risk-free zero coupon bond with maturity T and nominal $e^{r\,T}\,K$. By assumption, this trade can be set up at negative cost, so requires no funding. If the arbitrageur gets never exercised on the call option, at maturity the trade payoff equals

$$S_T + e^{r\,T}\,(K - S_T)_+ - e^{r\,T}\,K - (S_T - K)_+$$
$$= \min\{K, S_T\}\,\Big(1 - e^{r\,T}\Big),$$

which is non-negative by assumption $r \leq 0$. If the arbitrageur gets exercised on the call option at time $\tau \leq T$, her stock is delivered into the call, the remaining portfolio is held until maturity, with trade payoff

$$e^{r\,T}\,(K - S_T)_+ + K\,e^{r\,(T-\tau)} - K\,e^{r\,T} \geq 0,$$

again due to our assumption $r \leq 0$. This justifies (3).
Let (δ, Y) be an arbitrary model satisfying (i), (ii) and (iii). Using $r \leq 0$ and convexity of the function $s \mapsto (K-s)_+$, we find

$$\begin{aligned} P_0(K,T) &\overset{(r\leq 0)}{=} e^{-r\,T}\,\mathbb{E}\Big[\big(K - S_T\big)_+\Big] \\ &\geq e^{-r\,T}\,\big(K - \mathbb{E}[S_T]\big)_+ = \Big(e^{-r\,T}\,K - S_0\,e^{-\delta\,T}\Big)_+. \end{aligned}$$

This provides a lower bound on the put price for fixed δ, independent of the choice of Y. Now we assume that for some model $(\delta_u, \tilde{Y})$ this lower bound is attained (and not equal to zero), i.e. we assume that there is a model $(\delta_u, \tilde{Y})$ satisfying (i), (ii), and (iii), and

$$P_0(K,T) = K\,e^{-r\,T} - S_0\,e^{-\delta_u\,T}.$$

Then δ_u is a model-free upper bound on the repo rate, since

$$\begin{aligned} K\,e^{-r\,T} - S_0\,e^{-\delta_u\,T} &= P_0(K,T) \geq \Big(K\,e^{-r\,T} - S_0\,e^{-\delta\,T}\Big)_+ \\ &\geq K\,e^{-r\,T} - S_0\,e^{-\delta\,T}. \end{aligned}$$

Thus, $\delta_u \geq \delta$, as claimed. Left to verify is thus only that we can find such model $(\delta_u, \tilde{Y})$ under our assumptions.
Intuitively, we wish to define a stock price model $(\tilde{S}_t)$, induced by a pair $(\delta_u, \tilde{Y})$, with "maximal uncertainty" about the value of $\tilde{S}_\epsilon$ for infinitesimally small $\epsilon > 0$, given the observed prices $P_0(K,T)$ and $C_0(T,K)$. In the limit as $\epsilon \searrow 0$ this amounts to a model in which the observed number S_0 becomes a random variable. Concretely, we consider the stochastic process

$$\tilde{Y}_t = (S_0 + u)\,1_{\{B=1\}}, \quad t \geq 0,$$

where $u \geq 0$ is a constant and B is a Bernoulli-distributed random variable with success probability $S_0/(S_0+u)$. Apparently, the natural filtration $(\mathcal{F}_t)$ of $(\tilde{Y}_t)$ is trivial, and $(\tilde{Y}_t)$ is a constant function, hence a martingale, with respect to this filtration. We denote the American-style put and call prices within this easy model by $\tilde{P}_0(K,T)$ and $\tilde{C}_0(K,T)$, and observe that

$$\begin{aligned} \tilde{C}_0(K,T) &= S_0\,\frac{(S_0+u-K)_+}{S_0+u}, \\ \tilde{P}_0(K,T) &= \begin{cases} K\,e^{-r\,T} - S_0\,e^{-\delta_u\,T} & \text{, if } K\,e^{(\delta_u - r)\,T} \geq (S_0+u) \\ \frac{u}{S_0+u}\,K\,e^{-r\,T} & \text{, else} \end{cases}. \end{aligned}$$

In addition to the parameter δ_u, this model has the free parameter $u \geq 0$. A calibration of these two free parameters to the observed prices $C_0(K,T)$ and $P_0(K,T)$ is always possible and extremely simple, to wit

$$u = S_0\,\frac{K + C_0(K,T) - S_0}{S_0 - C_0(K,T)},$$
$$\delta_u = -\frac{1}{T}\,\log\Big\{\frac{K\,e^{-r\,T} - P_0(K,T)}{S_0}\Big\}.$$

Notice that the first equality for u is always possible to achieve, since the model-free bounds[3]

$$(S_0 - K)_+ \le C_0(K,T) \le S_0$$

hold. Plugging the definition of u that matches with $C_0(K,T)$ into the model put price, we obtain

$$\tilde{P}_0(K,T) = K\,e^{-r\,T} - \min\{S_0\,e^{-\delta_u\,T}, (S_0 - C_0(K,T))\,e^{-r\,T}\}.$$

We observe that in the free parameter $\delta_u \ge 0$ this yields put prices in the range

$$(K + C_0(K,T) - S_0)\,e^{-r\,T} \le \tilde{P}_0(K,T) \le K\,e^{-r\,T}.$$

While the upper bound is clearly a model-free bound satisfied by $P_0(K,T)$ under our assumption $r \le 0$, the lower bound holds because of (3). This finishes the argument. □

3.2 A candidate model attaining δ_ℓ

We consider a model $\tilde{S} = (\delta_\ell, \tilde{Y})$ with one-parametric model for $\tilde{Y}$ with closed form prices for American-style calls and puts, and which has a high chance that the call is never exercised. To this end, we assume

$$\tilde{Y}_t := S_0\,e^{\lambda\,t}\,1_{\{\tau > t\}}, \quad t \ge 0,$$

where τ is exponential with rate $\lambda \ge 0$. Due to the lack-of-memory property of the exponential law, $(1_{\{\tau>t\}})$ is a continuous-time Markov chain that changes its state only once at τ. This implies that the natural filtration of $(\tilde{Y}_t)$, which is assumed to be the market filtration $(\mathcal{F}_t)$, is given by

$$\mathcal{F}_t = \big\{\emptyset, \Omega, \{\tau > t\}, \{\tau \le t\}\big\}.$$

The motivation for this simple model in the present article originates from two sources. On the one hand, the model is simple enough to evaluate American-style prices in closed form, see Lemma 3.2 below. This makes it possible to decide very quickly, whether the early exercise premium of the call is positive, which is precisely what's needed to prove sharpness of the upper bound on $F(0,T)$ derived before. On the other hand, the model is motivated to some extent by reversing the intuition of the extreme model $(\delta_u, \tilde{Y})$ that was used in the proof of Theorem 3.1. Intuitively, in that model the stock price decreases deterministically, only at trade inception $t = 0$ there is a potential extreme spike. To reverse this logic, we now look for a model in which the stock price increases deterministically, only once at a later time

[3] The lower bound is simply the intrinsic value, and the upper bound follows from the fact that $(\exp(-r\,t)\,S_t)$ is a super-martingale.

point $t = \tau$ there is an extreme downward jump, and we hope that the market prices allow to expect τ to happen after T so that "no exercise" is the optimal strategy. It turns out that in our example for Wirecard AG presented in Section 2 this logic works out, with parameters being chosen as $\delta_\ell = 0.84$ and $\lambda = 3.0013$.

Lemma 3.2 (American-style call option price)
Let $K > 0$. The American-style call option price in the simple model $\tilde{S} = (\delta_\ell, \tilde{Y})$ is given by $\tilde{C}_0(K,T) = (S_0 - K)_+$, if $r + \lambda \leq 0$, and for $r + \lambda > 0$ by $\tilde{C}_0(K,T) =$

$$\begin{cases} (S_0 - K)_+ & , \delta_\ell > 0 \text{ and } \frac{S_0}{K} \geq u_C \\ \Big(S_0\, e^{-\delta_\ell\, T} - K\, e^{-(r+\lambda)\, T}\Big)_+ & , \Big(\delta_\ell = 0\Big) \text{ or } \Big(\delta_\ell > 0 \text{ and } \frac{S_0}{K} \leq \ell_C\Big) \\ \Big(S_0 \left(\frac{\delta_\ell\, S_0}{(r+\lambda)\, K}\right)^{\frac{\delta_\ell}{r+\lambda-\delta_\ell}} - K \left(\frac{\delta_\ell\, S_0}{(r+\lambda)\, K}\right)^{\frac{r+\lambda}{r+\lambda-\delta_\ell}}\Big)_+ & , \text{else} \end{cases},$$

where

$$u_C := \max\left\{\frac{1 - e^{-(r+\lambda)\,T}}{1 - e^{-\delta_\ell\, T}},\, \frac{r+\lambda}{\delta_\ell}\right\},$$
$$\ell_C := u_C \cdot \min\left\{1,\, e^{-(r+\lambda-\delta_\ell)\, T}\right\}.$$

Proof
Obviously, the only non-deterministic stopping time with respect to the market filtration is τ, but $\min\{\tau, T\}$ is clearly a suboptimal strategy for the call option (since it returns zero value in the case $\tau \leq T$). Hence, it is sufficient to maximize the function

$$f(t) := e^{-r\,t}\,\mathbb{E}\big[(\tilde{S}_t - K)_+\big]$$
$$= S_0\, e^{-\delta_\ell\, t} - e^{-(r+\lambda)\, t}\, K$$

in the deterministic stopping time $\tau = t$. To this end, we note that generally $\tilde{C}_0(0,T) = S_0$, and for positive strike $K > 0$ we distinguish the following cases:

(1) $\delta_\ell = 0$:

(1.1) $r + \lambda = 0$:
Obviously, $\tilde{C}_0(K,T) = f(0)_+ = f(T)_+$.

(1.2) $r + \lambda \neq 0$:
We see $f^{'}(t) < 0$ if and only if $r + \lambda < 0$, so that

$$\tilde{C}_0(K,T) = \begin{cases} f(0)_+ & , \text{if } r + \lambda < 0 \\ f(T)_+ & , \text{if } r + \lambda > 0 \end{cases}$$

(2) $\delta_\ell > 0$:

(2.1) $r + \lambda \leq 0$:
The function $f(t)$ is decreasing, so $\tilde{C}_0(K,T) = f(0)_+$.

(2.2) $0 < r + \lambda < \delta_\ell$:
If $(r + \lambda) \geq \delta_\ell\, S_0/K$, then f is non-decreasing and the call will not be exercised. Otherwise, the function $f(t)$ is decreasing for $t \leq t_*$ and increasing thereafter, where t_* is positive and given by

$$t_* := \frac{\log\left(\frac{K\,(r+\lambda)}{\delta_\ell\, S_0}\right)}{r + \lambda - \delta_\ell}.$$

This implies

$$\tilde{C}_0(K,T) = \begin{cases} \max\{f(0), f(T)\}_+ & , r+\lambda < \delta_\ell\, \frac{S_0}{K} \\ f(T)_+ & , \text{else} \end{cases}.$$

Since $g_T(x) := (1-\exp(-x\,T))/x$ is non-increasing in $x \geq 0$, we observe that

$$\frac{r+\lambda}{\delta_\ell} \leq \frac{1-e^{-(r+\lambda)\,T}}{1-e^{-\delta_\ell\,T}}.$$

Furthermore, $f(T) \geq f(0)$ if and only if

$$\frac{S_0}{K} \leq \frac{1-e^{-(r+\lambda)\,T}}{1-e^{-\delta_\ell\,T}}.$$

Summarizing, we obtain

$$\tilde{C}_0(K,T) = \begin{cases} f(T)_+ & , \frac{S_0}{K} \leq \frac{1-e^{-(r+\lambda)\,T}}{1-e^{-\delta_\ell\,T}} \\ f(0)_+ & , \text{else} \end{cases}.$$

(2.3) $r+\lambda = \delta_\ell$:
Obviously, $\tilde{C}_0(K,T) = f(0)_+$.

(2.4) $r+\lambda > \delta_\ell$:
The function $f(t)$ is increasing for $t < t_*$ and decreasing thereafter, where t_* is the same as in case (2.2). Furthermore, we have

$$t_* > 0 \Leftrightarrow r+\lambda > \delta_\ell\,\frac{S_0}{K},$$
$$t_* < T \Leftrightarrow r+\lambda < \delta_\ell\,\frac{S_0}{K}\,e^{(r+\lambda-\delta_\ell)\,T},$$

which implies

$$\tilde{C}_0(K,T) = \begin{cases} f(0)_+ & , r+\lambda \leq \delta_\ell\,\frac{S_0}{K} \\ f(T)_+ & , r+\lambda \geq \delta_\ell\,\frac{S_0}{K}\,e^{(r+\lambda-\delta_\ell)\,T} \\ f(t_*)_+ & , \text{else} \end{cases}.$$

Putting together all cases, we end up at the claimed formula. When summarizing the cases, it is helpful to make use of the relation

$$r+\lambda \geq \delta_\ell \;\Leftrightarrow\; \frac{r+\lambda}{\delta_\ell} \geq \frac{1-e^{-(r+\lambda)\,T}}{1-e^{-\delta_\ell\,T}},$$

which is true for $r+\lambda > 0$ (and $\delta_\ell \geq 0$), since $g_T(x)$ from case (2.2) is decreasing in $x \geq 0$. $\square$

4 Related open questions

We find the following open research questions related to the present article interesting:

1. If assumptions (i)-(iii) are satisfied for some model $S = (\delta, Y)$ in which the call is never exercised, do we necessarily find parameters δ_ℓ, λ such that the simple model $(\delta_\ell, \tilde{Y})$ of Section 3.2 also satisfies (i)-(iii) and the call is never exercised? In other words, is the model of Section 3.2 the right "extreme model" to be looked at in general, or have we been just lucky in our presented Wirecard AG example?

2. How to modify the presented logic in the case $r > 0$? This case appears to be more difficult, since the early exercise premium of the put option has value as well.

3. Let (δ, Y) be some model with one-parametric Y with parameter σ, e.g. the Black–Scholes model. From the observed prices $P_0(K,T)$ and $C_0(K,T)$, how to efficiently back out the two parameters (δ, σ) numerically? An idea is to implement the following generic algorithm: Start with $\sigma = 0$ and fit δ to the put price $P_0(K,T)$. This δ is maximal. If the call is not matched, then increase σ a little bit, again fit to $P_0(K,T)$. The found δ is a little smaller. Proceed like this until the call is matched as well. This idea utilizes that $P_0(K,T)$ is increasing in δ and σ, while $C_0(K,T)$ is decreasing in δ and increasing in σ. By this method, one obtains ultimately a model-dependent range for the forward that is narrower than the model-free bounds presented.

4. How to enhance the results to a situation with more observed prices for different strikes $K_1 < \ldots < K_n$? In particular, if prices for all (infinitely many) strikes $K \geq 0$ are observed, does the interval $[\delta_\ell, \delta_u]$ necessarily narrow down to a singleton?

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