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How does a repo margin affect credit-equity models?

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Summary The repo margin that a stock owner can earn by lending her stock influences the pricing of stock derivatives. This effect is especially pronounced for distressed stocks when the repo margin is non-negligibly high. Intuitively, one might interpret the repo margin as the market's opinion about the size of the negative drift of the stock price. This means if the repo margin is high, the market expects the stock to decline sharply. The present note explains how the repo margin enters credit-equity models which are used for detecting lucrative mispricings between credit and equity instruments of the same company (Section 2), and how it affects expected income computations (Section 3).

1 Introduction

Before we start, it is important to mention that by the term "repo margin" we mean an instantaneous rate $\delta \geq 0$ that is earned by someone who owns a stock. Technically, in order to earn it the stock owner must lend his stock in the marketplace via so-called repurchasement agreements, see Bernhart, Mai (2014) for details. Furthermore, it is assumed for the sake of a simplified notation that δ is a constant. In theory, it is also possible to earn a repo margin when possessing a bond, by lending the latter in the marketplace. However, this is explicitly excluded throughout the present note, which is the reason why we only speak of "repo margin" and do not explicitly specify "equity repo margin". Equivalenty, one may also interpret δ as a continuous dividend yield, which is another interpretation for the same mathematical quantity - namely a negative drift for the stock price process under a risk-neutral pricing measure.
Assume we observe the prices of a long credit instrument (e.g. a bond) and of a short equity instrument (e.g. a put). We build up a position consisting of both instruments with the intention to profit from the proceeds of the long credit instrument, while hoping that the short equity instrument minimizes losses in case of a worsening in the creditworthiness of the company under concern. Thus, the short equity instrument is considered as a hedge for the long credit instrument, and this hedge is expected to cost something (e.g. the put is likely to lose time value). By quantitative means, portfolio management decides whether the expected hedging costs of the short equity instrument, which have to be subtracted from the expected proceeds of the long credit instrument, are cheap enough in order for the overall position to be considered lucrative. In this context, the present article demonstrates why two related, quantitative figures are increasing functions of the repo margin assumption: On the one hand, the size of

the favorable mis-pricing between the long credit and the short equity instrument, and on the other hand the expected income of the whole position. These observations are counterintuitive on the first glimpse, since in theory a high repo margin makes the prices of the short equity instruments expensive, thus affecting the attractiveness of the overall position in a negative way. However, the controversy is resolved by observing that the model repo margin assumption only affects the future expectation of our short equity instrument in a favorable way, but the current price is independent thereof – simply because it is a fixed input observed in the marketplace. These findings are explained in a more elaborate way in the remaining sections of this document.

2 Repo margins and capital structure arbitrage detection

For the detection of a favorable mis-pricing between credit and equity instruments of some company, a so-called *capital structure arbitrage opportunity*, one might make use of so-called 1.5-*factor credit-equity models*. Before discussing the effect of the repo margin assumption on such an analysis in Subsection 2.2, we provide an intuitive description of 1.5-factor credit-equity models in Subsection 2.1.

2.1 Intuitive description of 1.5-factor credit-equity models

Without going into mathematical details, the intuitive idea of a 1.5-factor credit-equity model is as follows: As long as the company under consideration is solvent, its stock price is modeled as a one-factor diffusion process, e.g. like in the Black–Scholes model or in a more general local volatility model[1]. When the company becomes insolvent, the stock price jumps to zero and remains there eternally. The random future time point of the company's insolvency, the so-called *default time*, is modeled in such a way that – before default actually occurs – the likelihood of a default within the next instant of time stands in a reciprocal relationship with the current stock price. In other words, the likelihood of a sudden default is low when the stock price is high, and vice versa. In mathematical terms, this intuitive model is defined on a probability space with probability measure $\mathbb{P}$, sometimes called the *historical (or statistical) probability measure*.
General Finance theory states that such a financial market model is free of arbitrage opportunities if and only if there is a probability measure $\mathbb{Q}$, a so-called *risk-neutral pricing measure*, such that (a) the measure $\mathbb{Q}$ is equivalent[2] to the original probability measure $\mathbb{P}$ of the model, and (b) under $\mathbb{Q}$ the discounted wealth process resulting from a portfolio which is long one stock is a (local) martingale, see, e.g., Jarrow et al. (2007). "The discounted wealth process being a martingale" intuitively means that an investor is indifferent between investing his or her money into a risk-free bank account (specified by the discounting rate) or into the risky stock, because both strategies – or any combination of the two – are expected to earn the same profit under $\mathbb{Q}$. In this arbitrage-free case, the fair price of any financial instrument, whose payoff(s) depend on the stock price and/or the timing of

[1] See, e.g., Mai (2015) for background on local volatility models.

[2] This means intuitively that any event that can happen with respect to the original probability measure $\mathbb{P}$ can also happen in the model with the new probability measure $\mathbb{Q}$.

the default event, is given by the expected and discounted value of the respective payoff(s), where the expectation must be computed with respect to one of the aforementioned risk-neutral pricing measures, a concept known as *risk-neutral valuation*. Deviation from this concept introduces arbitrage, i.e. the resulting prices can be exploited to earn a risk-free profit which is higher than the one generated by the risk-free bank account. It is important to notice that the risk-neutral pricing measure needs not be unique, but the resulting prices are invariant with respect to the choice of a particular risk-neutral pricing measure. Consequently, required for the pricing of derivatives within a financial market model is the knowledge about the probability distribution of the default time and the stock price under some (arbitrary) risk-neutral pricing measure.

This being said, one can show mathematically that typical 1.5-factor credit-equity models are free of arbitrage and for pricing derivatives within such a model it is required to find one risk-neutral pricing measure. Indeed, (Bielecki et al., 2011, Proposition 2.1) shows that the aforementioned investor's indifference between risk-free and risky investment is accomplished within the 1.5-factor credit-equity model when the drift of the stock price process before default is proportional to the default intensity (which itself is dependent on the stock price) under $\mathbb{Q}$. Intuitively, under $\mathbb{Q}$ szenarios in which the credit-risk of the company grows significantly coincide with the szenarios in which the stock price rallies before default actually occurs. This appears counterintuitive, as we might model the stock price under $\mathbb{P}$ rather with a bearish trend in case of a low creditworthiness, but the counterintuitive risk-neutral pricing measure $\mathbb{Q}$, which was just described, is essentially unique (and thus the only admissible pricing measure) when assuming that CDS are traded as well, cf. (Bielecki et al., 2011, Proposition 2.1).

2.2 Repo margin and mis-pricing detection

The influence of the model parameters in a typical 1.5-factor credit-equity model on the fair prices of credit and equity instruments is visualized in Figure 1. One important observation to be made from Figure 1 with regards to the topic of the current article is that the repo margin δ does not enter the pricing of pure credit instruments. This is crucial for understanding how the repo margin is chosen when calibrating the model to observed market prices of credit and equity derivatives. Observed prices of pure credit instruments might be used in order to calibrate all model parameters except the repo margin. Given all these fitted parameters, the observed equity derivative price can be compared with a model-implied price for the derivative, under the additional, exogenous assumption of a repo margin. If the model price turns out higher than the market-observed price, then there is a favorable mis-pricing between credit and equity that one might consider for an investment. A typical situation is that one hedges a long credit position with an equity derivative that has a short-equity character, such as a put option or an equity forward in which one commits oneself to sell the stock at a fixed price in the future.

However, if the equity derivative in concern has a short equity

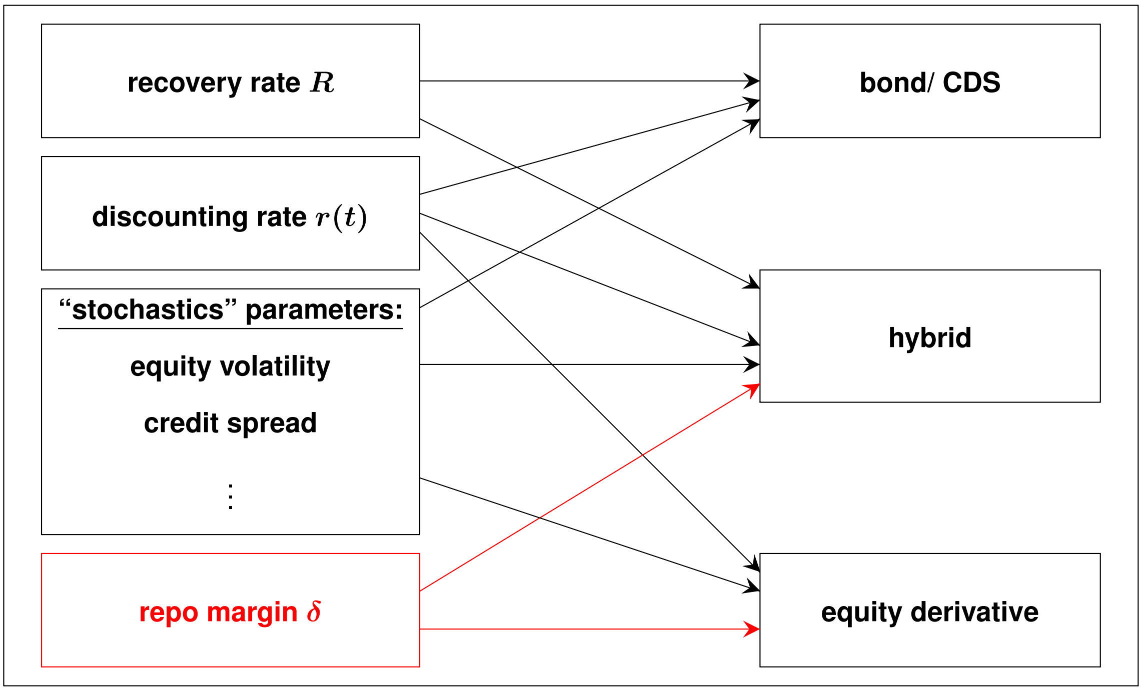

Fig. 1: Visualization of how the model parameters in an exemplary jump-to-default credit-equity model affect prices of credit and equity derivatives. It is important to recognize that the repo margin does not affect the pricing of CDS or regular bonds at all.

character, its model price is increasing in the repo margin. This is because a higher repo margin goes along with a lower expectation about the level of the future stock price. Consequently, the potential mis-pricing under consideration becomes more favorable the higher the assumed repo margin. On the first glimpse, this appears counterintuitive, because the repo margin is something that has to be paid continuously when shorting the equity, hence seemingly making the investment less attractive. However, the given market quote for the equity derivative already comes with an intrinsic repo margin assumption – made by the trader that provides the offer. If our model assumption for δ is higher than the intrinsic assumption in the quote, then our model price is overestimated and the mis-pricing appears more favorable than it actually is. Admittedly, a higher repo margin assumption also implies a higher roll cost assumption in case the short equity position needs to be prolonged in the future, e.g. in case of a maturity mismatch between the credit investment and the hedging equity investment. The latter roll costs, however, are difficult to take into account appropriately – and they are not taken into account when only comparing the currently quoted prices of instruments with maturity mismatch. Consequently, when screening market quotes of credit and equity instruments for inconsistencies it is important to apply a realistic repo margin assumption. If an estimate for the repo margin is difficult (or not available at all), a conservative assumption is $\delta = 0$, because this implies the smallest possible model price for derivatives with short equity character.

3 Repo margins and expected income computation

The presence of a favorable mis-pricing between credit and equity instruments of a company does not immediately yield a numeric value for how much money one is expected to earn above risk-free when entering into the model-implied arbitrage. The estimation of such an expected income, we also call it *carry*, is discussed in the present section. We pursue a carry computation on an instrument level, i.e. for each and every position in a single instrument we compute a carry. Generally speaking, the carry of an equity derivative over a time period of length[3] Δ is computed as the expected PnL of the derivative, i.e.

$$\begin{aligned}\text{Carry} \quad &= \quad \text{expected value (at } t=\Delta)\\ &\quad - \quad \text{current value (at } t=0).\end{aligned}$$

The crucial step in computing the expected carry is to compute the expected value of the derivative at $t = \Delta$. It is crucial to understand that this computation relies heavily on an assumption of how the stock price will turn out at Δ. The model-free, risk-neutral expectation for the value of the stock price at Δ is called the Δ-*Forward*. Under the absence of dividend payments within $[0, \Delta]$, the Δ-Forward is given by

$$F(0,\Delta) := \mathbb{E}^{\mathbb{Q}}[S_\Delta] = S_0\, e^{(r-\delta)\,\Delta},$$

where S_0 equals the current stock price and r denotes the (for simplicity constant) risk-free interest rate. Consequently, the (risk-neutral) expected future value of the stock already depends on the repo margin. Since the repo margin constitutes a deterministic downward drift of the stock price, a higher repo margin implies a lower expectation about the future stock price. Consequently, we expect the carry measurements of equity derivatives with a “short equity-character” to be increasing in the repo margin. Indeed, this can easily be verified for a specific instrument.
Exemplarily, we consider an equity forward contract in which we commit ourselves to sell the stock in the future. Denote the forward's deal strike price by K and its maturity by T, and assume for simplicity that $T \geq \Delta$. Expecting that the stock price at Δ will be at the level $F(0,\Delta)$, the expected value of the forward at $t = \Delta$ equals

$$\begin{aligned}&e^{-r\,(T-\Delta)}\left(K - \mathbb{E}^{\mathbb{Q}}[S_T\,|\,S_\Delta = F(0,\Delta)]\right)\\ &\quad = e^{-r\,(T-\Delta)}\left(K - \underbrace{F(0,\Delta)\,e^{(r-\delta)\,(T-\Delta)}}_{=:F(0,T)}\right),\end{aligned}$$

and the current value of the forward equals

$$e^{-r\,T}\left(K - F(0,T)\right) = e^{-r\,T}\left(K - S_0\,e^{(r-\delta)\,T}\right).$$

Using these formulas and simplifying the terms, the expression for the forward carry becomes

$$\text{Forward carry} \quad = \quad \left(e^{r\,\Delta} - 1\right)\left(K\,e^{-r\,T} - S_0\,e^{-\delta\,T}\right).$$

As expected, it follows that the forward carry is an increasing function in the repo margin δ.

[3] In our case, typically Δ = one month.

From the equity forward example we encounter the same counterintuitive observation as in Section 2: shouldn't the carry be a decreasing function of the repo margin, since we continuously pay δ when shorting the stock? In theory, if the forward contract was entered into at time $t = 0$ the fair forward deal strike K in the previous derivation should equal precisely $F(0, T)$, in particular should depend on the repo margin δ, so that the carry equals zero. In other words, the deal strike K of the forward comes with an intrinsic repo margin assumption, which is made by the forward counterparty. If our model assumption for δ differs from this intrinsic assumption, then the forward carry is negative or positive. Keeping the quoted offer K fixed and increasing our model repo margin assumption, the carry increases, because our model assumptions about the future decline of the stock price are more favorable as those implied by the market quoted deal strike K.

4 Conclusion

We have demonstrated why two related, quantitative figures in the context of a capital structure arbitrage position are increasing functions of the repo margin assumption. On the one hand, the size of the favorable mis-pricing between a long credit and a short equity instrument, and on the other hand the size of the expected income of a position consisting of both the long credit and the short equity instrument. These observations are counterintuitive on the first glimpse, since in theory a high repo margin makes the prices of the short equity instruments expensive, thus affecting the attractiveness of the overall position in a negative way. However, the controversy is resolved by observing that the model repo margin assumption only affects the future expectation of our short equity instrument in a favorable way, but the current price is independent thereof – simply because it is a fixed input parameter which is observed in the marketplace.

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