



**JOINTLY HEDGING
JUMP-TO-DEFAULT RISK AND
MARK-TO-MARKET RISK:
SOME CONSIDERATIONS**

Jan-Frederik Mai
XAIA Investment GmbH
Sonnenstraße 19, 80331 München, Germany
jan-frederik.mai@xaia.com

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Abstract When entering a position that is long credit, two major sources of risk are present. On the one hand, there is the fundamental jump-to-default (JTD) risk, which is most important from a hold-to-maturity perspective. On the other hand, there is mark-to-market (MTM) risk due to daily fluctuations in the credit spreads. When hedging such a long credit position by a short equity position, both risk constituents have an effect on the hedge ratio and need to be balanced. The overall position should be JTD-neutral, meaning that default risk is hedged perfectly, and MTM-neutral, meaning that MTM risk is hedged perfectly. When hedging with a long put position, the choice of the strike price provides enough freedom to make the overall position both JTD- and MTM-neutral, which in general is not possible with a pure (linear) forward hedge.

1 Two sources of risk Assume we set up a capital structure arbitrage position with the idea to be long credit (risk) of a certain company A, and this risk is hedged via a position that is short the equity of A. The long credit (risk) position may be realized by buying a bond issued by A or by selling CDS protection on the underlying reference entity A. The short equity position may be realized by buying put options on the stock of A or by entering a forward contract with the obligation to sell the stock of A at a future point in time for a pre-determined strike price.

Loosely speaking, when entering a position that is long credit risk of company A, two major sources of risk are present:

(a) *Jump-to-default (JTD) risk*. The risk that a credit event related to company A is triggered, and hence the long credit position suffers a severe, sudden loss. This is the fundamental risk of the position. If we were pure hold-to-maturity investors without the need to care about anything else than receiving promised cash flows from our long credit position, this was the only risk we would have to care about. In order to hedge JTD risk, one needs to know how the position is affected by a credit event. Since the precise effect is unknown a priori, several approximations have to be made. Firstly, a credit position typically does not lose all of its value upon a default event, since the bankrupt company's property is distributed among creditors, resulting in a so-called recovery value of the position. We have to make an assumption on the recovery value that is relevant for our specific credit instrument, because it is needed to specify the amount that is actually at



stake. Secondly, in order to set up a short equity hedging position, we must make an assumption on how it behaves upon a credit event. One realistic assumption in many situations is that the stock price jumps to zero (or almost zero). Given this assumption, the value of a forward contract to sell the stock at a pre-determined strike price, as well as the value of a put option on the stock, equals precisely the strike price upon a default event. Hence, the loss given default of the hedging equity position equals that amount minus the current position value. Summarizing, the JTD-neutral hedge ratio is basically given by the formula

$$\begin{aligned} & \text{Number of forwards/puts required for JTD-neutrality} \\ &= \frac{\text{loss given default of the credit position}}{\text{loss given default of the equity position}} \end{aligned}$$

In particular, this formula depends on the strike price of the forward/put. If a forward is used as a hedging instrument, then the strike price is given by the market and there is no choice. If puts are used as hedging instrument, however, the market might offer puts with different strikes, which provides us with a choice to pick the one that is most convenient for us.

- (b) *Mark-to-market (MTM) risk.* The risk of a loss in the position that is due to a change of the market's opinion about company A's creditworthiness. These changing opinions can be tracked on a daily basis by monitoring the fluctuations of credit spreads/ market prices. The resulting MTM losses (gains) per se do not alter any promised cash flows from our credit investment, they only indicate that the probability for receiving future cash flows decreases (increases). Consequently, they matter if we want to get out of the position and require a buyer for it in the marketplace. In order to hedge mark-to-market risk, one needs a mathematical model which relates mark-to-market movements of the hedging equity position with mark-to-market movements of the credit position. The canonical models capturing this effect are so-called 1.5-factor credit-equity models, see Mai (2012) for an introduction. The underlying idea is to model the stock price exogenously by some stochastic process, e.g. like in a Black-Scholes type model. This model for the stock price can then be used to compute an equity-delta of the hedging position. Moreover, the company's credit spread is assumed to be given by a decreasing function of the stock price (i.e. stock \uparrow implies credit spread \downarrow , and vice versa). Hence, the credit position can also be assigned an equity-delta within such a framework. Consequently, the hedge ratio which makes the overall position MTM-neutral, i.e. invariant with respect to daily market fluctuations, is approximately given by

$$\begin{aligned} & \text{Number of forwards/puts required for MTM-neutrality} \\ &= - \frac{\text{equity-delta of the credit position}}{\text{equity-delta of the hedging equity position}} \end{aligned}$$

Similar as in (a), the equity-delta of the hedging equity position depends on the strike price of the applied put options. If



the hedge is set up with put options (rather than forwards), this provides us with the additional freedom to pick the strike of our choice.

2 An illustrative example

Let us consider an exemplary capital structure arbitrage position on the valuation date 26-Jul-2013. The credit position consists in selling CDS protection on the reference entity A. More precisely, we have a position of nine CDS contracts, in which we sell protection, maturities ranging between March 2017 and June 2020, with a total nominal of 33 Mio USD and a running coupon of 500 bps. Currently, the value of this CDS position is approximately equal to -1.4755 Mio USD. In order to hedge the risk of this position, we would like to buy put options on the stock of company A. The current stock price is 3.82 USD, and we buy put options maturing in January 2015. The questions we face are: (i) How many puts do we have to buy? (ii) Which put strikes should we choose? In the sequel we provide answers to these questions, applying the aforementioned criteria (a) JTD-neutrality and (b) MTM-neutrality.

(a) *JTD-neutrality.* Upon a default event we lose the CDS position value of -1.4755 Mio USD, i.e. we make a MTM-gain. Moreover, we have to compensate the insurance buyer(s) for their losses, which amounts to a payment of $(1 - R) 33$ Mio USD, with recovery rate R . Assuming a recovery rate of $R = 40\%$, our total loss given default on the CDS position is thus approximately equal to 18.3245 Mio USD. Assuming we have N puts on the underlying stock with strike price K USD, and also assuming that the stock price jumps to zero upon a default event, the gain we make on the put position equals $N \cdot K$ USD minus N times the market value of the put $P(K)$. This implies that the optimal number $N^{JTD}(K)$ of put options with strike price K in order to be JTD-neutral is

$$N^{JTD}(K) = \frac{18.3245}{K - P(K)} \text{ million.}$$

Notice that the required prices $P(K)$ are typically not all observable in the marketplace, but only for some strikes K . Therefore, a model must be applied in order to interpolate or extrapolate the whole function $K \mapsto P(K)$ for all strike levels K from the few quoted prices.

(b) *MTM-neutrality.* We observe a CDS credit curve for company A, to which we calibrate a 1.5-factor credit-equity model, so that it explains the observed CDS quotes pretty well, see Figure 1. Given the model specified in this way, we can compute an equity-delta for our CDS position, which is visualized in Figure 2 and denoted by ΔC in the sequel. One observes that an upward (downward) move in the stock implies a gain (loss) in our CDS position, but the relationship is non-linear. Similarly, the same model can be used to compute an equity-delta for one put option with strike price K . We denote the value of the put option by $P(K)$ and its equity-delta by $\Delta P(K)$. If we buy N put options with strike K our overall position, consisting of the CDS position and the put hedge, has

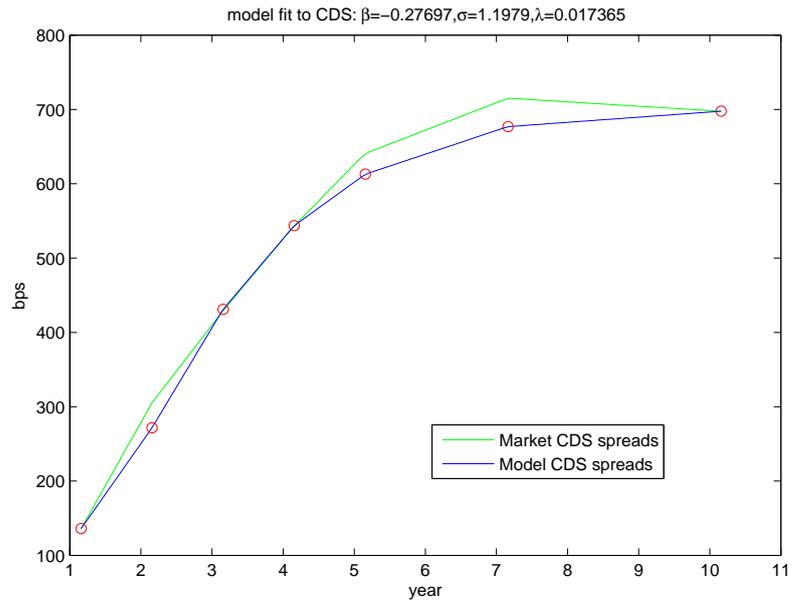


Fig. 1: Fit of a three-parametric specification of the JDCEV credit-equity model introduced in Carr, Linetsky (2006) to observed CDS spreads on 26-Jul-2013.

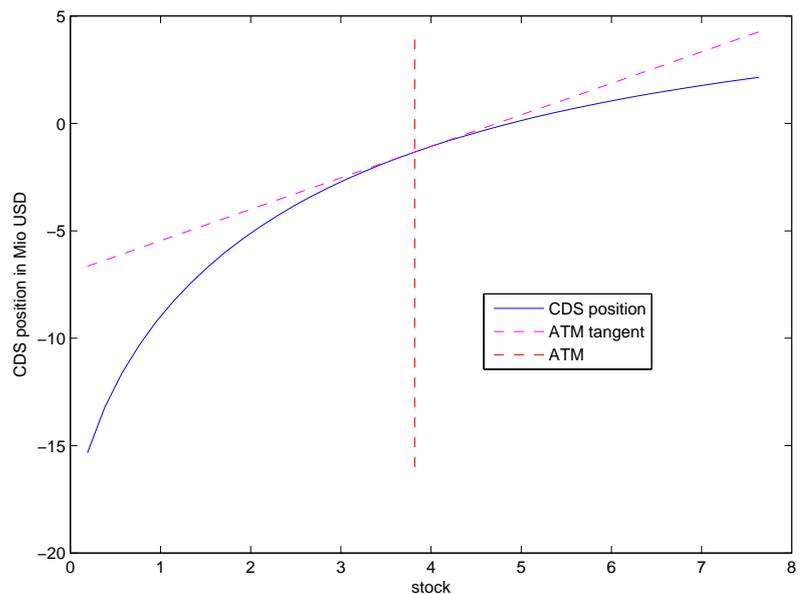


Fig. 2: Sensitivity of the short CDS position with respect to shifts in the stock price, computed with the fitted credit-equity model.

an equity delta of $\Delta C + N \cdot \Delta P(K)$. In order to be MTM-neutral, this delta must be zero yielding the optimal number $N^{MTM}(K)$ of puts given by the formula

$$N^{MTM}(K) = -\frac{\Delta C}{\Delta P(K)}.$$

Figure 3 visualizes the functions $K \mapsto N^{JTD}(K)$ and $K \mapsto N^{MTM}(K)$. Both functions are decreasing in the strike price K of the put options. For $N^{JTD}(K)$ this is clear because the value

of the option upon default is simply increasing in the strike price. For $N^{MTM}(K)$ the decreasing nature is explained by the fact that the sensitivity of the put option on movements in the underlying stock price (the equity-delta) is increasing. For instance, it is intuitive that a far-out-of-the-money put is hardly affected by a small movement of the underlying stock. It is observed – at least in the present example – that for far out of the money put options MTM-neutrality requires a higher hedge ratio than JTD-neutrality, and that this statement is reversed for in-the-money put options. What are the practical implications we can derive from Figure 3? For example, if we decide to hedge with put options that have strike price 3 USD, then we would need approximately 10 Mio put options in order to be JTD-neutral. However, Figure 3 indicates that this hedge ratio overestimates the MTM-risk. The sweet spot strike level K^* , for which we have JTD- and MTM-neutrality, i.e. $N^{JTD}(K^*) = N^{MTM}(K^*)$, is approximately 1.2415 USD (32.5% moneyness) in this example.

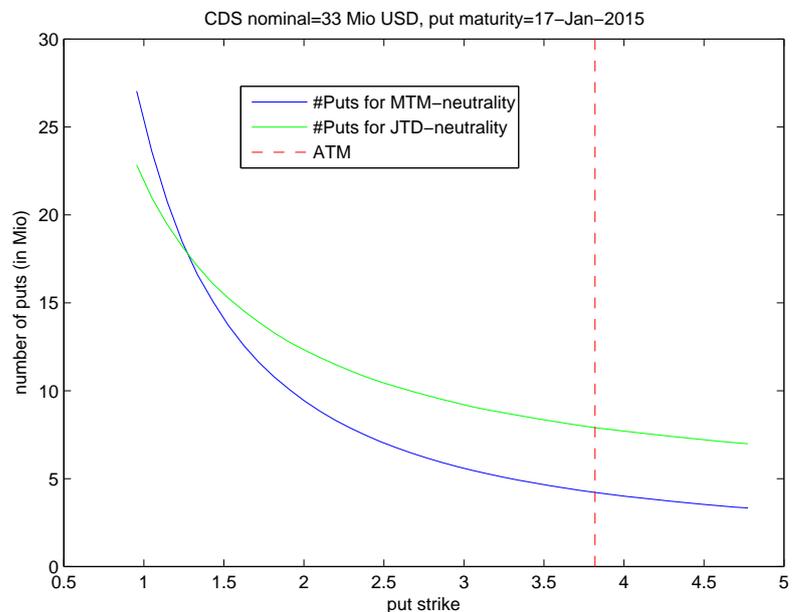


Fig. 3: Optimal number of put options required in order to be MTM-neutral and JTD-neutral, in dependence on the strike price. It is observed that the optimal strike level, where the blue and green lines intersect, lies at approximately 32.5% moneyness.

3 Some practical issues

Clearly, the presentation so far has been ignoring several real-world constraints that have to be addressed when actually entering such a position. In the sequel, we briefly point the reader at two of these issues.

3.1 Availability of strike level

What if the analysis indicates the use of a 50% moneyness strike level, but in the marketplace only 80% moneyness (or more) are available? Moreover, even if many strike levels are available, one might encounter situations in which the bid-ask prices for the different strike levels differ significantly. Such liquidity restrictions add another dimension to the optimization problem of



“choosing the optimal strike level” and must be addressed on a case-by-case level. A further criterion that might play a role in choosing the strike level is one’s personal view towards the likelihood of a default event, and whether one wishes to be JTD-positive or JTD-negative (rather than JTD-neutral).

3.2 Computation of deltas One must keep in mind that the required equity-deltas for MTM-neutrality are highly model-dependent¹. The resulting hedge ratios might differ when different models are applied. For instance, for the equity-delta of the stock options, should we use a credit-equity model or the market standard Black-Scholes model? Or is it important that the equity position and the credit position are evaluated jointly with one credit-equity model, from which the respective deltas must be computed? If so, what if no credit-equity model can explain both market prices jointly, because there is a relative mispricing (which is the desired case in a capital structure arbitrage position)? The quantification of such model risk is a non-trivial issue and requires a certain amount of experience.

4 Conclusion It was illustrated that the optimal choice of a strike level when hedging a long credit position via a short equity position is a challenging task. Several criteria that play a dominant role for this decision have been highlighted, both from an analytical and from a liquidity point of view.

References J.-F. Mai, The joint modeling of debt and equity: an introduction, *XAIA homepage article* (2012).
P. Carr, V. Linetsky, A jump to default extended CEV model: an application of Bessel processes, *Finance and Stochastics* **10:3** (2006) pp. 303–330.

¹Some of the put prices $P(K)$ required for the computation of the JTD-neutral hedge-ratio are also model-dependent, since there are strike prices for which no market quotes are observable.