



HEDGING CALLABLE BONDS WITH CREDIT DEFAULT SWAPS

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Abstract The cash flows of a callable bond depend on the issuer's decisions in the future, hence are unknown today. Consequently, hedging the default risk of such a bond with a maturity-matched credit default swap (CDS) bears the risk that the CDS becomes orphaned after an early call and needs to be closed with a significant loss. A practical solution might be to split the CDS hedge into several maturity buckets, covering the range of potential call dates. We describe in simple terms how the allocation of the CDS hedge into different maturity buckets can be achieved, based on the computation of market-implied probabilities that the issuer exercises his or her call rights.

1 Motivating example First of all, it is very important to be aware of the fact that a call feature of a bond can make it more costly to hedge away its default risk via CDS. In Appendix B, we provide an explicit example of a financial market model with a callable bond and a CDS that is arbitrage-free, but which is no longer arbitrage-free in case one ignores the call feature of the bond. In other words, a call feature of a bond in general can kill a potential arbitrage opportunity that might be spotted when ignoring the call feature. This tiny example already suggests that finding a CDS hedging strategy for a callable bond can be a very difficult exercise in general. In order to motivate this challenge further, we begin with a concrete numeric example that is inspired by a real-world case. We consider a EUR-denominated bond that pays a coupon rate of 5% and matures in five years in April 2020 (valuation date is in February 2015). The issuer has the right to call the bond on any time after two years according to the following schedule:

from	until	call right
now	April 2017	not callable
April 2017	April 2018	callable at strike 105%
April 2018	April 2019	callable at strike 102.5%
April 2019	maturity	callable at strike 100%

We assume that we buy 1 million EUR nominal of this bond at a market price of 106.5%, i.e. we spend 1.065 million EUR. With the intention to reduce our default risk we intend to buy CDS protection, for the sake of simplicity¹ also with nominal value 1 million EUR. The question we deal with in the present article is: which CDS maturity is appropriate? In order to sharpen the readers' awareness about the non-triviality of this question, we

¹In reality, the total CDS nominal is typically determined by considering the so-called *Jump-To-Default* measurement, see Appendix A for details.



briefly sketch what might happen in the two most obvious CDS maturity choices:

(a) **What if we hedge to the first call date?**

Since the bond currently trades at a level above the first call strike in two years, it is not unlikely that the bond gets called at the first possible call date in two years. Consequently, one strategy could be to buy only two year CDS protection. The advantage of this decision is that one saves the money for CDS protection between first call and maturity date. However, what if the creditworthiness of the bond issuer decreases dramatically within the next two years, but no credit event takes place? In this case the bond value drops significantly and the issuer's incentive to call the bond in two years fades away. Consequently, the CDS hedge has become worthless after two years and we need to prolong the hedge. However, buying CDS protection in two years for the bond's remaining lifetime has become very expensive because of the worse creditworthiness. Similarly, selling the bond results in a loss on the position, since its value might be far below its value at trade inception.

(b) **What if we hedge to maturity?**

In order to prevent the adverse scenario in bulletpoint (a) above another strategy could be to buy CDS protection until the bond's maturity. The advantage of this decision is that default risk is practically eliminated. However, the drawback of such a decision is that this hedge might be quite expensive. Moreover, what if the bond gets called in two years? Then we are left with a CDS contract whose remaining lifetime is another three years, but the initial intention of the CDS as hedge for the bond is lost. Even worse, it is highly likely that the market value of this CDS drops dramatically in case of a call, because either the CDS orphans fully (because all eligible bonds have been called), or the call of the bond is the result of a significantly improved creditworthiness resulting in a drop of the CDS protection value.

This example and the considerations in (a) and (b) above emphasize the need to split the CDS hedge into different maturity buckets. But how much CDS nominal should be allocated to which maturity? One obvious idea², which is demonstrated below, is to determine this maturity allocation based on market-implied call probabilities. This means that the higher the call probability in two years the bigger the share of two year CDS protection, and vice versa. The bond can be called on any date after two years. This means that all available CDS insurance contracts with lifetime longer than two years are potential hedge candidates. Taking into account the fact that the bond's maturity is five years and that available CDS maturities are quarterly, this leaves 13 possible CDS maturities across which we must allocate a total of 1 million EUR CDS nominal: June 2017, September 2017, ..., March 2020, and June 2020. In accordance with the

²Other choices may be reasonable as well, e.g. taking care of delta-neutrality with respect to parallel shifts in the CDS curve etc..



aforementioned idea to determine the maturity allocation based on call probabilities, we need to assign 13 call probabilities, namely the probability to get called within the lifetime of the June 2017 CDS, the probability to get called within the lifetime of the September 2017 CDS but not the June 2017 CDS, the probability to get called within the lifetime of the December 2017 CDS but not the September 2017 CDS, and so on. These 13 probabilities sum up to unity, if one defines “ordinary redemption at maturity” as “getting called at maturity”, which is equivalent from an economic viewpoint. Consequently, we can multiply any of these 13 probabilities with our total CDS nominal 1 million EUR to compute the nominal required in the respective maturity bucket, leaving us with precisely 1 million EUR CDS nominal in total.

2 What is the use of such a hedge?

Sticking with the example from the previous section, assume we have assigned (by any means) the following call probabilities: with 50% probability, the issuer calls the bond within the lifetime of the June 2017 CDS, and with 50% the issuer does not call the bond at all, so that we need the June 2020 CDS. In this case, what is the use of a hedge position that assigns .5 million EUR to the two year and .5 million EUR to the five year CDS hedge? In any case, half of our CDS hedge exhibits a loss potential caused by one of the scenarios (a) or (b) described in the previous section. Of course, this also certainly saves us half of the loss in the adverse scenarios described in (a) and (b), but still it does not sound really satisfying. In particular, one has to be aware that such a position is “credit long”, compared with a maturity-matched position. The usefulness of the hedge only becomes visible in case the call probabilities are re-adjusted on a regular basis, and if the call probability for a certain maturity bucket tends to either the value 1 or 0 when the end of the respective maturity bucket approaches. If this is satisfied, then a regular re-adjustment of the hedge leaves one with a perfectly balanced portfolio at all time points – although being transaction costly, and although potentially exhibiting negative PnL in case that an approaching call probability tends to zero and the hedge needs to be prolonged. In particular, the aforementioned “coin toss-uncertainty” between call or no call should not occur briefly before the first call date. Instead, one should by then have a rather good idea about whether a call is imminent or not. However, the well-behavedness of the call probabilities over time in practice is contingent on rational behavior of the issuer. By construction, there is an information asymmetry between bond issuer and market participants because the former knows about his or her call intention whereas the latter do not know. However, if the bond issuer bases his or her call decision on rational behavior this means that a call will be beneficial if the current bond value is above the prevalent call strike, a condition that market participants can observe via the market price and whose probability they can track over time. If the issuer decides to call (not to call) the bond at a strike price above (below) its current market value, typical financial market models fail to predict this appropriately.



3 Assigning call probabilities

Callable bonds are typically priced by a *backwardation algorithm* involving a discrete-time approximation of the so-called *default intensity*, which is a mathematical object describing the issuer's (changing) creditworthiness over time, cf. (Schönbucher, 2003, Chapter 7.4) for an excellent introduction. The parameters of such a model can be calibrated to observable market prices of the issuing company's bonds and/or CDS contracts referencing on the issuer. The pricing algorithm can be described in intuitive terms as follows: at the bond's maturity there is a finite number of possible scenarios for the then prevailing default intensity, or the bond issuer is already in default. For each possible scenario, the price of the bond at maturity is known, namely it equals the final redemption and coupon payment in case of survival, and the recovery payment in case of default. The backwardation algorithm now works backwards in time. At the penultimate time step, again there is a finite number of possible scenarios for the default intensity, or the issuer is already in default. For each possible scenario the issuer can now make a call decision: either the bond is called at the then prevailing call strike. Or, if cheaper for the issuer, the bond is not called. The issuer's decision is based on a comparison of the call strike with the so-called *continuation value* of the bond. The latter equals the expected value of the bond at the penultimate time step in case the bond is not called, conditioned on the current level of the default intensity. Intuitively, if the default intensity is very high, then the continuation value is rather low, and vice versa. Consequently, there will be a number of scenarios for which the bond is called and its price will be equal to the call strike, and a number of scenarios for which the bond is not called and its price will be equal to the continuation value. Additionally, the bond value equals a recovery payment in the default scenario. Having assigned a bond value to each possible scenario, one may then proceed with the antepenultimate step, and so on. Iteratively working backwards in time, one ultimately computes the current bond value.

As a side product of such a pricing algorithm, one may also compute the desired call probabilities. Recall that at each time step for each possible scenario for the default intensity one has to make a call decision. Remembering these decisions within the pricing algorithm and working backwards in time, at the end of the backwardation one has essentially counted all possible scenario paths leading to a certain call decision, and hence has computed the probability for this call decision. For illustrational purposes, Figure 1 visualizes the call probabilities that have been computed in this way for our exemplary bond with maturity in five years in April 2020. The parametric model for the default intensity has been chosen such that it resembles a real-world business case. The remaining lifetime of the bond has been discretized to a weekly time grid. Figure 1 then shows the probability for a call decision at each single time step in this grid. One can observe that the call probability is equal to zero before the first call date in April 2017. Then there is a 38% call probability at the first call date. Moreover, there is a certain time period after the first call date, in which there is still a certain probability to get called, followed by a time period before April 2018, when the call proba-

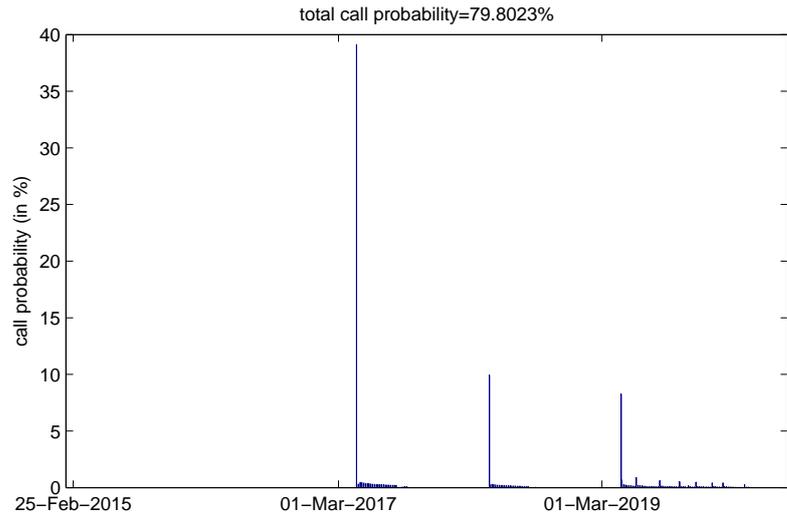


Fig. 1: Model-implied call probabilities for the exemplary bond.

bility is (almost) zero again. The call probability then has another spike in April 2018, when the call strike is reduced from 105% to 102.5% for the first time. The same pattern then repeats in April 2019. Summarizing, the probability that the bond is called during its lifetime (at some date) is 79.8%, most likely being a call within the first call period.

Remark 3.1 (Sensitivity of call probabilities w.r.t. volatility)

There is one important observation regarding model risk when computing call probabilities in the aforementioned way: these probabilities depend critically on the variance of the default intensity. In particular, there may be various models for the default intensity which all explain a given CDS curve equally well, but which exhibit different levels of variance. That is because the given CDS curve determines only the market’s current opinion about survival probabilities, but not the degree of fluctuation of this opinion over time. As an extreme example, think of a deterministic (e.g. piecewise constant) default intensity fitted perfectly to a given CDS curve, leading to all call probabilities being either 0 or 1. Any “true” stochastic model for the intensity fitted to the same curve is likely to imply a few call probabilities within the interval $(0, 1)$, like in Figure 1 above. In particular, when modeling the default intensity as an explosive diffusion³ its variance and consequently the resulting call probabilities might be significantly different from call probabilities resulting from a non-explosive diffusion, simply because in the further case it is much more likely that a path of the underlying bond price decreases dramatically, leading to a “non-call decision” by the issuer.

In order to illustrate the effect of volatility further Figure 2 shows 20 potential bond price simulations for an exemplary callable bond maturing in about 4.5 years⁴. The ten trajectories in the upper plot are simulated from a model which implies a lower variance of the bond price trajectories than the variance of the model

³See, e.g., Andreasen (2001).

⁴Another bond than the one from the previous example.

underlying the ten simulations in the lower plot. However, both model specifications are chosen such that they imply the same CDS curve. In the upper plot the total call probability is about 12.5%, whereas in the lower plot it is about 56%. The 4.5-default probability is about 37% in both model specifications, reflecting the fact that both models imply the same CDS curve.

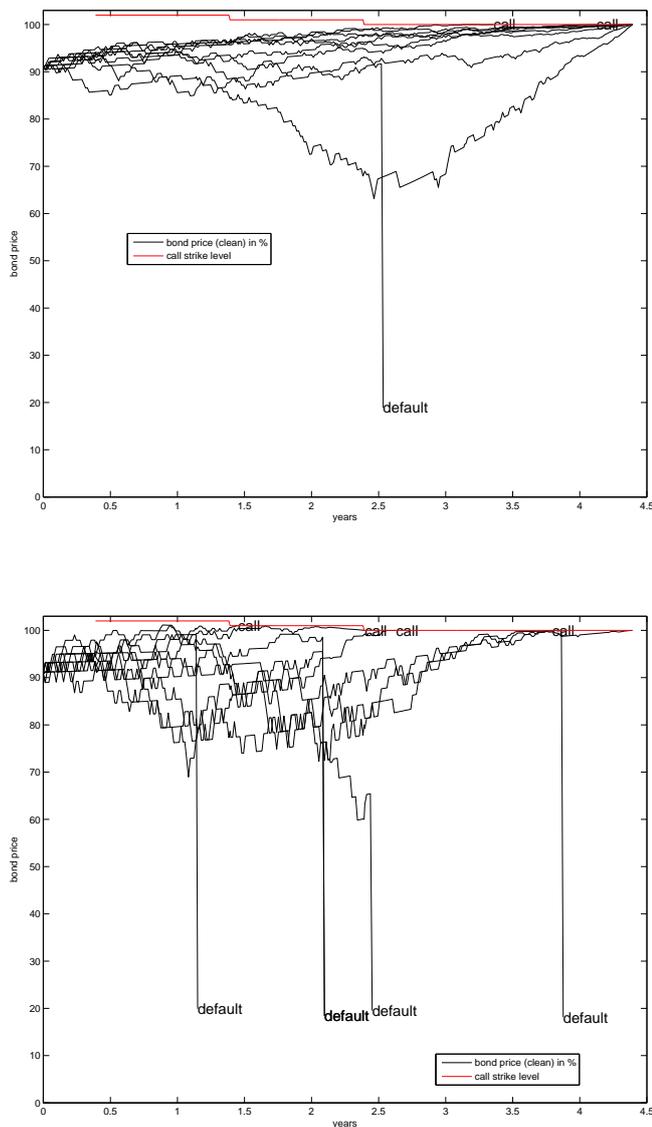


Fig. 2: Monte carlo simulations for possible future bond price trajectories. Top: model specification implying moderate variance for the bond price trajectories. Bottom: model specification implying high variance for the bond price trajectories.

4 Conclusion It was highlighted that the appropriate CDS-hedging of a callable bond is non-trivial in general. Moreover, it was demonstrated how market-implied call probabilities can be crucial quantities for the allocation of an appropriate CDS hedge. Finally, it was sketched by which means such call probabilities can be computed.



Appendix A: How to determine the (total) CDS nominal?

Let T_1, \dots, T_m be available CDS maturities, and $T_0 := 0$. The preceding sections show how to compute probabilities p_1, \dots, p_m , where

$$p_k := \text{probability that bond gets called within } (T_{k-1}, T_k],$$

for $k = 1, \dots, m$. Assuming that the bond's ultimate maturity is smaller than T_m , it follows that $p_1 + \dots + p_m = 1$. Given a total CDS nominal, say N_{CDS} , it has been argued that a reasonable hedging strategy is to split the CDS protection into m maturity buckets as follows: for each $k = 1, \dots, m$ buy CDS protection with maturity T_k and CDS nominal $p_k N_{CDS}$. A valid question, that has not been addressed in the main body of this article, is: what is a reasonable choice for N_{CDS} ? As pointed out in Mai (2013), the so-called *Jump-To-Default* is a quantitative figure that can be used to provide an answer. Intuitively, the approach is to choose N_{CDS} in such a way that a default within the next second (causing a loss on the bond and a gain on the CDS protection) is PnL-neutral. For the probability-weighted hedge described above, the Jump-To-Default $JTD(N_{CDS})$ is computed as

$$JTD(N_{CDS}) = \underbrace{N_{Bond} (R_{Bond} - B)}_{\text{bond loss}} + \underbrace{\sum_{k=1}^m p_k N_{CDS} (1 - R_{CDS} - \text{Upf}_k)}_{\text{CDS gain}},$$

where N_{Bond} denotes the bond nominal, R_{Bond}, R_{CDS} denote the recovery rates on bond and CDS, B is the current bond price, and Upf_k is the market value of a CDS with maturity T_k (aka its upfront payment). Postulating that the hedged position is Jump-To-Default-neutral means postulating $JTD(N_{CDS}) = 0$. In turn, this determines the total CDS nominal as

$$N_{CDS} = \frac{B - R_{Bond}}{\sum_{k=1}^m p_k (1 - R_{CDS} - \text{Upf}_k)} N_{Bond}.$$

In particular, it is observed that N_{CDS} depends on the call probabilities, as well as a recovery assumption.

Appendix B: Callability of a bond can kill the negative basis trade!

We construct a very simple financial market model. We assume zero interest rate for simplicity. Besides the risk-free bank account, we assume that the following three assets are traded.

- A two-year bond that is callable in one year at par and pays annual coupon rate of 5%. It comes at a price of 80%.
- A one-year CDS with annual insurance spread of 10%. It comes at a price (i.e. upfront) of $\frac{750}{51}\% \approx 14.71$.
- A two-year CDS with annual insurance spread of 10%. It comes at a price (i.e. upfront) of $\frac{10605}{1428}\% \approx 7.43\%$.

We assume that the recovery rates of bond and CDS in case of a default event are identical and equal zero. Furthermore, we model the uncertain price evolution of these assets via a two-period

financial market model (one period represents one year) on a discrete probability space with state space $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ consisting of only four possible outcomes:

- ω_1 = default in the first period at $t = 0.5$,
- ω_2 = credit spread widening but no default in first period,
no default in second period,
- ω_3 = credit spread widening but no default in first period,
default in second period at $t = 1.5$,
- ω_4 = strong credit spread tightening in first period, no default
risk anymore in second period.

In this tiny model, default happens either at $t = 0.5$ (on $\{\omega_1\}$), $t = 1.5$ (on $\{\omega_3\}$), or never (on $\{\omega_2, \omega_4\}$). In case of a default, we assume that the CDS insurance buyer has to pay the accrued coupon of 5%, which is half of the annual insurance spread, consistent with the assumption of a default at $t = 0.5$, respectively $t = 1.5$, i.e. in the middle of the respective insurance period. Figure 3 illustrates the possible price paths of the instruments that we model.

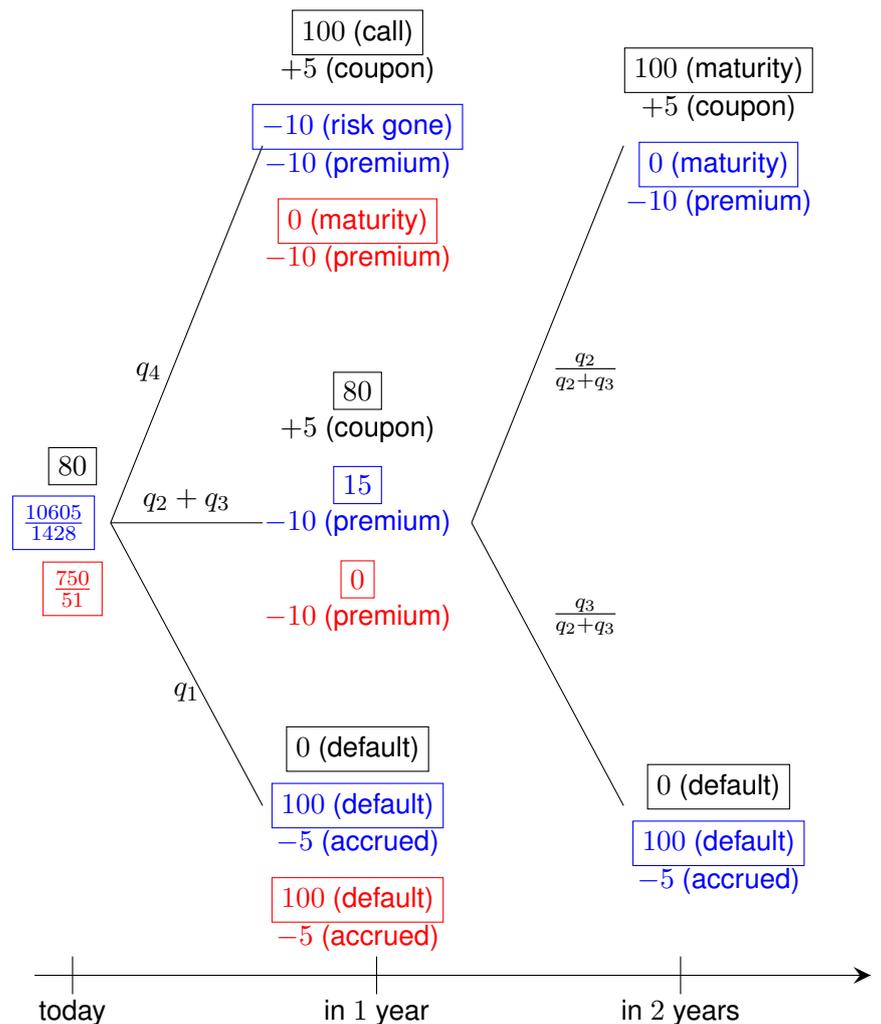


Fig. 3: Illustration of the possible paths for the bond price process (black), the price process of the one-year CDS (red), and the price process of the two-year CDS (blue).



The values q_1, \dots, q_4 in Figure 3 represent the unknown probabilities for scenarios $\omega_1, \dots, \omega_4$. All prices in Figure 3 are purposely chosen such that the financial market model is free of arbitrage. This can be seen by checking oneself that the choice

$$(q_1, q_2, q_3, q_4) = \left(\frac{4}{17}, \frac{4}{357}, \frac{5}{1428}, \frac{3}{4} \right)$$

yields a risk-neutral probability measure, i.e. the price processes of our three traded assets are all martingales with respect to this probability measure. Consequently, there exists no negative basis in this example.

However, we are now going to point out that there would be negative basis if we knew with certainty whether the bond was called or not. To this end, assume first that we knew the bond was called at time $t = 1$. In this case, there was no need for us to buy two-year CDS protection in order to protect ourselves from a loss in our bond investment. We could invest into the bond with nominal 100 at a price of 80 and buy nominal- and maturity-matched CDS protection at a price of $750/51$. On the event $\{\omega_1\}$ this implies a positive PnL of $-(80 + 750/51) + 95 = 15/51 \approx 0.29$. On the complementary event $\{\omega_2, \omega_3, \omega_4\}$ it implies a positive PnL of $-(80 + 750/51) + 105 - 10 = 15/51 \approx 0.29$. Now assume that we knew for sure the bond was not called. In this case, buying a maturity- and nominal-matched basis package implies an arbitrage. To see this, assume that we invest 100 into the bond and buy two-year CDS protection with nominal 100, no investment into one-year CDS. In this case, the potential PnL equals $-(80 + 10605/1428) + 95 = 10815/1428 \approx 7.57$ on the event $\{\omega_1\}$, and $-(80 + 10605/1428) + 90 = 3675/1428 \approx 2.57$ on the complementary event $\{\omega_2, \omega_3, \omega_4\}$. Since all these potential PnL's are positive, we have found an arbitrage. Notice in particular that the latter arbitrage opportunity vanishes in case the bond is callable, because on the event $\{\omega_4\}$ the payoff of the bond is only 105 due to the call, instead of 110 in the non-callable case.

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