XAIA INVESTMENT

When does a macro hedge improve portfolio performance?

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Abstract Within a traditional Markowitz-setting, it is investigated whether it is possible to improve the performance of a portfolio by adding a macro hedge to it. The result is demonstrated by an application to our fund XAIA Credit Curve Carry on 31 January 2017.

1 Introduction

Suppose there is a tradable basket I of assets, which we call an *index* in the sequel[1]. We denote the future PnL of the index within the next year by R_I, which is a random variable. Furhermore, suppose we have a portfolio P that is formed by selecting assets from the basket. The future PnL of that portfolio within the next year is denoted by R_P, another random variable. We are interested in whether we can optimize our portfolio by adding a macro-hedge via short-selling I. Mathematically, we seek to find optimal portfolio weights α, β such that the portfolio $\alpha\,P - \beta\,I$ results in an alternative portfolio that outperforms the original portfolio P with respect to a reasonable performance measurement. We assume that $\alpha > 0$ (i.e. we are long the portfolio P), but we allow for $\beta \in \mathbb{R}$, with negative values corresponding to being long the index and positive β corresponding to being short the index. Section 2 solves the problem in a traditional Markowitz setting by measuring the performance in terms of the fraction of expected PnL and standard deviation. Section 3 concludes.

2 Expected PnL divided by standard deviation

We assume that the first two moments of R_I, R_P exist and denote

$$\mathbb{E}[R_I] =: \mu_I > 0, \quad \text{Var}[R_I] =: \sigma_I^2 > 0,$$
$$\mathbb{E}[R_P] =: \mu_P > 0, \quad \text{Var}[R_P] =: \sigma_P^2 > 0.$$

Despite the current negative interest rate regime, the assumption of positive expected returns for most risky assets is still reasonable and typically satisfied in practice. Furthermore, we denote the Pearson correlation coefficient of R_I and R_P by $\rho \in [-1, 1]$. In the traditional Markowitz theory, cf. Markowitz (1952, 1959), any portfolio is described by the pair of its expected PnL and the associated variance, no other metrics are considered. Consequently, it is reasonable to prefer portfolio A over portfolio B if the expected PnL of portfolio A is higher than the expected PnL of portfolio B, while at the same time its variance is lower. However, this induces only a partial ordering to the set of all portfolios. To be able to order all portfolios we need a mapping from the space of all possible portfolios to $\mathbb{R}$ which increases in the expected

[1] We think of an equity index such as the DAX, or a credit default swap index such as the CDX IG, or a bond index such as the iBoxx, etc..

PnL and decreases in the variance, a *performance measure*. An obvious choice is the fraction of expected PnL divided by standard deviation. In our particular case, we seek to maximize the function

$$(\alpha,\beta)\mapsto \frac{\mathbb{E}[\alpha\,R_P-\beta\,R_I]}{\sqrt{\mathrm{Var}[\alpha\,R_P-\beta\,R_I]}},\quad \alpha>0,\,\beta\in\mathbb{R}.$$

It is observed that this function actually depends on its two variables (α,β) only through the fraction $u:=\beta/\alpha$. More specifically, it is given by

$$\frac{\mathbb{E}[\alpha\,R_P-\beta\,R_I]}{\sqrt{\mathrm{Var}[\alpha\,R_P-\beta\,R_I]}}=\frac{\mu_P-u\,\mu_I}{\sqrt{\sigma_P^2+u^2\,\sigma_I^2-u\,2\,\rho\,\sigma_I\,\sigma_P}}=:f(u).$$

The following can be shown about f.

Lemma 2.1 (Optimal index hedge)
We distinguish two cases:

(a) $\frac{\mu_P}{\sigma_P}\leq\rho\,\frac{\mu_I}{\sigma_I}$:
The function f achieves its supremum for $u\mapsto-\infty$.

(b) $\frac{\mu_P}{\sigma_P}>\rho\,\frac{\mu_I}{\sigma_I}$:
The function f achieves its maximum at $u_*:=\frac{\rho\,\mu_P-\frac{\mu_I}{\sigma_I}\,\sigma_P}{\frac{\mu_P}{\sigma_P}\,\sigma_I-\rho\,\mu_I}$.

Proof
This result may be obtained by computing the classical optimal Markowitz portfolio for two assets (P and I) with zero interest rate. Alternatively, direct computation shows that

$$f^{'}(u)\leq 0\Leftrightarrow u\,(\mu_P\,\sigma_I^2-\rho\,\sigma_I\,\sigma_P\,\mu_I)\geq\rho\,\sigma_I\,\sigma_P\,\mu_P-\mu_I\,\sigma_P^2. \tag{1}$$

In case (b), this shows

$$f^{'}(u)\leq 0\Leftrightarrow u\geq\frac{\rho\,\mu_P-\frac{\mu_I}{\sigma_I}\,\sigma_P}{\frac{\mu_P}{\sigma_P}\,\sigma_I-\rho\,\mu_I}=u_*,$$

which immediately implies the claim. In case (a), it is not difficult to observe with the help of Equation (1) that f achieves its supremum for $u\to-\infty$. □

The interpretation of the lemma is as follows.

Remark 2.2 (Interpretation of Lemma 2.1)

(i) If the considered performance measure for P is already less than or equal to ρ times the considered performance measure for I (case (a)), then we should put all our money into the index I and nothing into our portfolio P at all.

(ii) In the complementary case (b), which is both more interesting and more usual, there is an optimal hedge ratio $\beta/\alpha=u_*$ which can be negative (i.e. buy the index in addition to the portfolio) or positive (i.e. short-sell the index, as a hedge, as initially motivated). The larger the difference between the considered performance measure for P and the considered performance measure for I, the more favorite becomes shorting of the index.

(iii) If the correlation ρ is negative, we buy the index in both cases. This is intuitive, since going long the index adds diversification in this case.

(iv) If ρ (is positive and) satisfies

$$\frac{\mu_P}{\sigma_P} > \rho\,\frac{\mu_P}{\sigma_P} > \frac{\mu_I}{\sigma_I} > \rho\,\frac{\mu_I}{\sigma_I},$$

then $u_* > 0$, i.e. addition of an index macro hedge to the portfolio improves the performance measurement.

2.1 Application

We consider the fund XAIA Credit Curve Carry (XCCC) as portfolio P on 31 January, 2017. The fund consists of 17 positions, each of which comprises two credit default swaps (CDS) with different maturities. In each position we sell protection (so-called *short CDS*) in the longer-dated CDS and buy protection (so-called *long CDS*) in the shorter-dated CDS (same nominals), resulting in a curve flattener trade without default risk but with spread risk. Due to the maturity mismatch of the two CDS each position has a “long character”, i.e. the position is expected to suffer losses in case of a spread widening and to make a profit in case of a spread tightening. If the CDS curve remains unchanged within next year, we expect a “roll-down”-gain in case the CDS par spread curve is upward sloping, which is the base scenario defining our expectation. The average maturity of our long-dated short CDS lies between four and five years, while the average maturity of our short-dated long CDS lies between six months and one year. The CDS on the 17 reference entities are selected from a universe comprising different indices, namely the CDX EM, the CDX IG, the CDX HY, and the iTraxx XOVER, and we associate with each name in the portfolio P one of these indices. According to the respective weightings, we define as benchmark macro index I a portfolio consisting of a weigthed sum of short CDS with maturity five years on these four indices. Obviously, this benchmark position also has a “long character”, i.e. is expected to profit from a spread tightening and to suffer from a spread widening. Based on historical data[2] for the last year (262 data points), we estimate the required standard deviations as $\sigma_P = 5.64\%$ and $\sigma_I = 5.75\%$. The historical correlation of I and P estimated from the same data is given by $\rho = 71.86\%$. Instead of estimating the required expected returns historically, we compute them under the assumption of unchanged CDS curves within the next year (base scenario), resulting in an expected roll-down gain of $\mu_P = 5.32\%$, as well as[3] $\mu_I = 5.19\%$ (resp. $\mu_I = 5.36\%$). Based on these numbers, Figure 1 visualizes the function f from Lemma 2.1. It is observed that it is not favorable to add a macro hedge to the portfolio, instead it would even be favorable to add a short CDS position in the index, whose nominal is approximately 73% of the current portfolio's total CDS nominal. However, the improvement in performance is rather small.

[2] Provided by S&P capital.

[3] The expected return measurement μ_I in this example includes transaction costs, so it makes a difference whether β is positive or negative. While 5.19% is the expected return when selling five year protection on the index, the same scenario assumptions imply that it costs 5.36% to buy five year protection on the index, the difference being bid/ask.

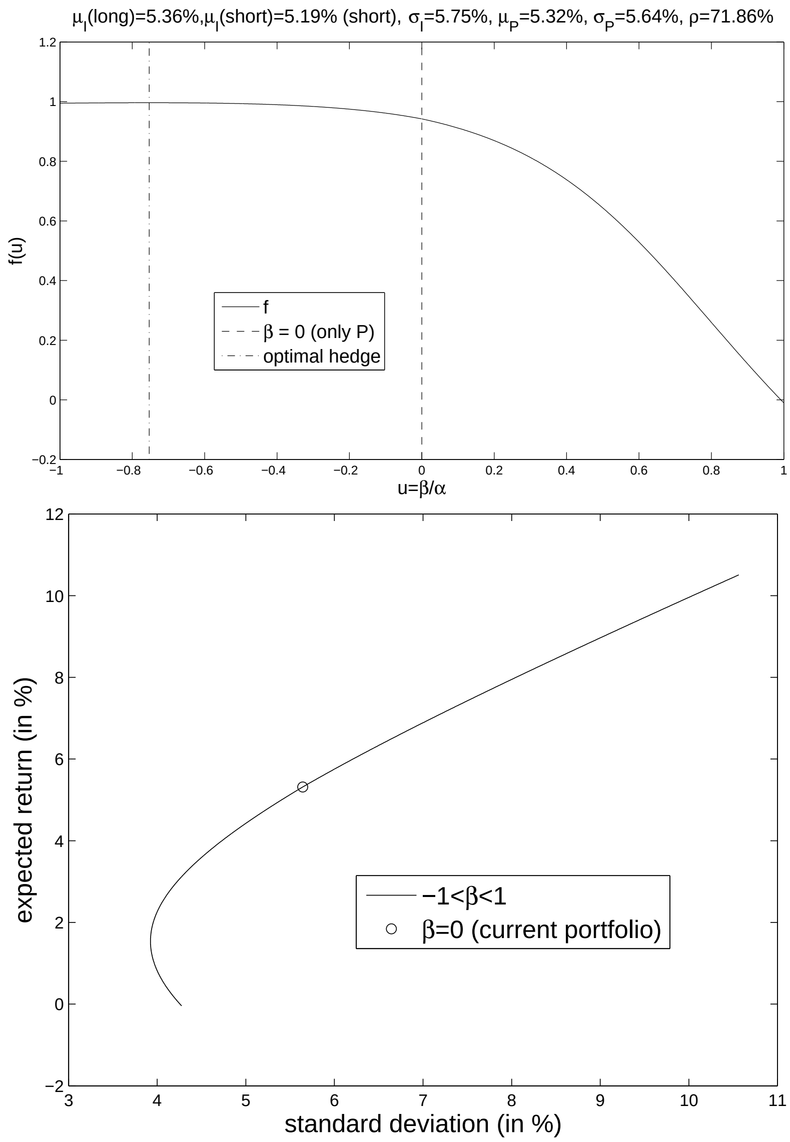

Fig. 1: Top: Performance of a portfolio consisting of XCCC and a position in the benchmark index, which only buys (or sells for negative u) five year protection. Bottom: Visualization of the numerator and denominator of f in dependence of $\beta \in [-1, 1]$ (for fixed $\alpha = 1$).

Figure 2 visualizes the same function f, only the benchmark index is defined slightly different. Instead of only selling five year CDS protection, for each index we additionally buy one year CDS protection, so that also on the index level we consider a curve flattener position as benchmark. It is observed that the estimated return and benchmark of the index change to $\mu_I = 3.24\%$ (resp. $\mu_I = 3.43\%$) and $\sigma_I = 5.03\%$, whereas the correlation with our portfolio remains almost unchanged at $\rho = 71.93\%$. In this second case, the portfolio P is already optimal.

The results for the two benchmark indices differ strongly because the considered performance measurement is significantly lower in case of the curve flattener benchmark. This is basically due to the fact that the short-dated CDS in that case significantly lowers the expected return μ_I of the index, while it has only little effect

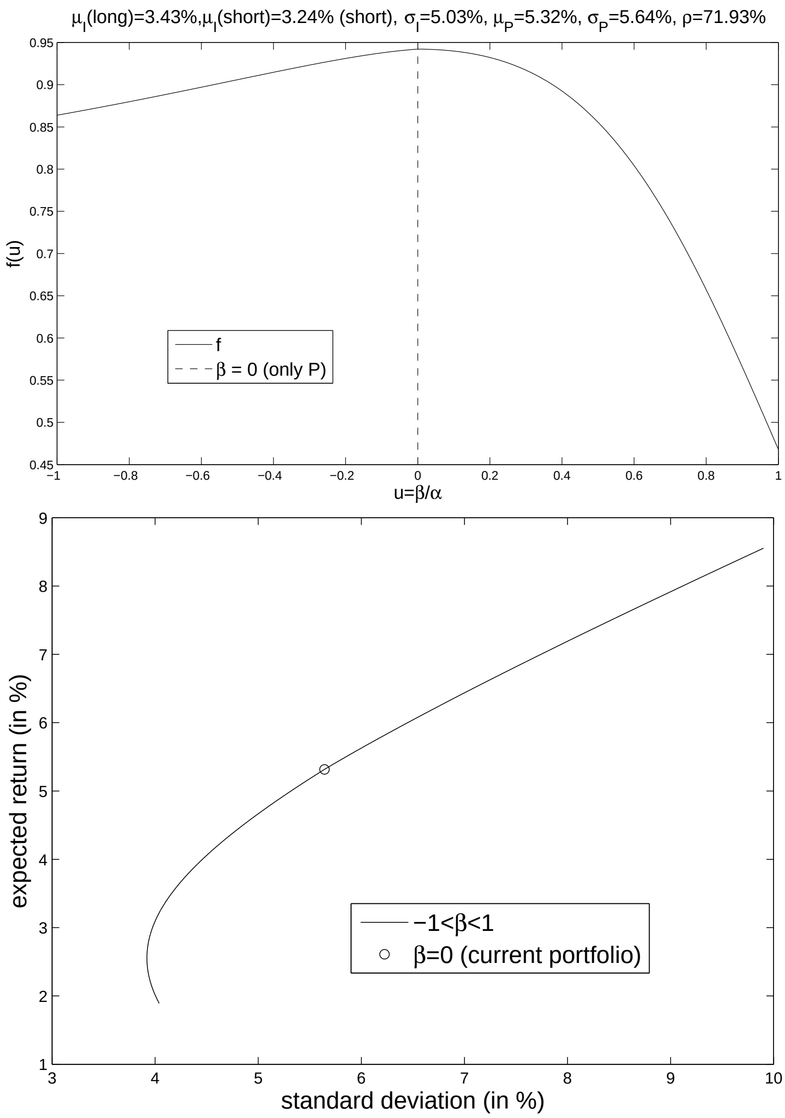

Fig. 2: Top: Performance of a portfolio consisting of XCCC and a position in the benchmark index, which buys five year protection and sells one year protection (or sells five year protection and buys one year protection for negative u). Bottom: Visualization of the numerator and denominator of f in dependence of $\beta \in [-1, 1]$ (for fixed $\alpha = 1$).

on the standard deviation σ_I. There is one potential explanation for this: The one year index CDS are not liquidly traded, and the applied price data for its computation is obtained by extrapolation of the index CDS curve that typically starts at the three year point. This extrapolation method over time seems to imply one year CDS upfront evolutions that are unrealistically smooth, so do not increase the volatility of the index enough.

3 Conclusion

Using a classical Markowitz-setup, it has been demonstrated under which circumstances it makes sense to add a macro hedge, in terms of shorting a benchmark index, to an existing portfolio. This is the case if the correlation coefficient ρ of the portfolio with the benchmark index is positive and the benchmark performance is smaller than ρ times the portfolio performance. As an application, it was demonstrated that our fund XCCC was not in need of a macro hedge by the end of January 2017.

References

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