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On model uncertainty in credit-equity models

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Abstract Credit-equity models are often used to infer equity derivative prices from observed prices of credit instruments referring to the same company, or vice versa. There is a huge degree of model freedom, hence model uncertainty, when doing this. The introduction of reasonable model axioms that diminishes this model uncertainty is more art than science. The present note investigates this model uncertainty and aims to provide a feeling for the effect of commonly made assumptions.

1 Introduction

Throughout, we denote by $r(.)$ a determinsitic, risk-free short rate used for discounting cash flows, and we denote by $\delta(.)$ a deterministic, continuous dividend yield associated with the stock of a company XY. The share price at time $t \geq 0$ is denoted by S_t, and the first time point in the future when a credit event with respect to company XY occurs is denoted by τ. We consider mathematical models that are capable of pricing both credit instruments referring to company XY and equity derivatives with the share of XY as underlying. In accordance with financial theory, this means that the considered models induce a probability space on which both $S = \{S_t\}_{t \geq 0}$ and τ are defined, with (risk-neutral) probability measure $\mathbb{Q}$ having the property that the wealth process of the portfolio that is long one stock is expected to accrue at the risk-free rate $r(.)$ on average. Intuitively, under $\mathbb{Q}$ an investor is indefferent between investing into the risky asset S or into the risk-free bank account. Mathematically, the non-negative process

$$e^{-\int_0^t r(s)-\delta(s)\,\mathrm{d}s}\, S_t, \quad t \geq 0, \tag{1}$$

needs to be a $\mathbb{Q}$-local martingale with respect to the market filtration, cf. Delbaen, Schachermayer (1994, 1998). Given such a model, the price of a European-style equity derivative with maturity $T > 0$ and payoff function h is given by the expectation of $h(S_T)$ with respect to $\mathbb{Q}$, multiplied with the discount factor $\exp(-\int_0^T r(s)\,\mathrm{d}s)$, explicitly

$$\mathsf{ED}(h,T) := e^{-\int_0^T r(s)\,\mathrm{d}s}\, \mathbb{E}_{\mathbb{Q}}\big[h(S_T)\big]. \tag{2}$$

American-style and more exotic equity derivatives in general depend on the probability distribution of the whole path $\{S_t\}_{t \in [0,T]}$ under $\mathbb{Q}$. What about credit derivatives? Having made an assumption about the recovery rate R in a potential auction following a credit event, the price of a credit default swap (CDS) within such a model is given in terms of the function $t \mapsto \mathbb{Q}(\tau > t)$, i.e.

is determined by the risk-neutral default probabilities, explicitly

$$\text{CDS}(R,c,T) := (1-R)\int_0^T e^{-\int_0^t r(s)\,\mathrm{d}s}\,\mathrm{d}\mathbb{Q}(\tau \leq t) - c\int_0^T \mathbb{Q}(\tau > t)\,e^{-\int_0^t r(s)\,\mathrm{d}s}\,\mathrm{d}t, \tag{3}$$

where c denotes the contractually specified CDS running premium. Furthermore, under the assumption that the value of a bullet bond issued by company XY jumps down to the recovery value R at τ, its price is given by a similar formula. Like in the case of equity derivatives, in case of more exotic credit derivatives, e.g. like bonds with call options, the respective pricing formulas depend on the evolution of survival probabilities over time[1].

One popular use of a credit-equity model is to infer the prices of equity derivatives from the ones of credit derivatives, or vice versa. For example, if one observes market prices for CDS with different maturities, it is possible to calibrate a credit-equity model to these observations making use of formula (3). If the CDS observations are sufficient to determine all parameters of the model, the calibrated model can be applied to generate model prices for European equity derivatives via formula (2). The latter might now be compared with actually observed equity derivative prices in the marketplace to make a decision whether the available equity derivatives are cheap relative to the credit derivatives. If this is the case, either the market offers an opportunity for a relative value trade, or the market is efficient and the model is unrealistic. If one believes in an efficient market, one major criterion for checking the quality of a credit-equity model is its capability of jointly explaining all observed market prices for credit and equity derivatives. However, the following issues point out that “fitting capability” is not everything in practice:

(i) A second important criterion that is in direct conflict with the capability of explaining all prices is the requirement for a **small number of model parameters** in order to maintain tractability and to reduce identifiability issues. This results in the classical trade-off between realism and tractability which every theoretical model faces when it is applied, and which often turns applied mathematics more into art than science.

(ii) Some market participants believe that the **efficient market hypothesis does not hold**. Instead, based on empirical research and expert opinion, they believe in certain relationships between credit derivative and equity derivative prices that they postulate as model axioms. Put differently, they trust the projections of their models more than the observed market prices.

(iii) There are **fundamental relationships between credit and equity** of a company that cannot be ignored. It is of course easy to define a credit-equity model that is consistent with finance theory (i.e. which induces a risk-neutral probability

[1] The function $t \mapsto \mathbb{Q}(\tau > t)$ provides the survival probabilities at time $t = 0$, but at $t > 0$ with more market information revealed, the whole function might fluctuate.

measure) and explains all observed market prices. One may simply model the equity and credit components of the model completely independent of each other, i.e. assume that τ and S are independent under $\mathbb{Q}$. One may then construct some arbitrary model for τ that explains all observed credit derivatives, and a model for S that explains all credit derivatives. For both tasks there is extensive literature and numerous models with good fitting capacities can be found. However, this independence assumption is neither justified empirically (by observing historical prices of equity and credit derivatives) nor fundamentally, since the valuations of equity and debt of a company are clearly interrelated.

As a consequence of (iii) above, it is necessary to make modeling assumptions on the relationship between credit and equity in the model. In the present article, we investigate some specific credit-equity models, satisfying one or more of the following axioms that narrow down the modeling cosmos in increasing order:

(A1) **Jump-to-default assumption:**
At time τ it is assumed that the share price jumps to zero and remains there, i.e.

$$\mathbb{Q}(\tau \leq t) = \mathbb{Q}(S_t = 0), \text{ for all } t > 0.$$

This assumption reflects the idea that the company goes bankrupt at time τ, the company XY is handed over from the equity holders to the creditors, and after all debt is serviced no equity is remaining.

(A2) **Credit spread and share price are reciprocal:**
We assume (A1) and, additionally: if the share price falls (rises), CDS prices increase (decrease). This assumption reflects the idea that the default scenario associated with assumption (A1) does not only hold at τ, but is actually anticipated in market prices also before τ. This is an assumption that can often be observed empirically, see, e.g. Figure 1.

(A3) **Market anticipates credit event:**
We assume (A2) and, additionally: between today and the occurence of a credit event at τ there must be at least one time period with significantly increasing CDS prices (resp. decreasing share price), so that τ cannot happen completely out of the blue. This assumption incorporates the empirical observation that a credit event typically follows a time period with extremely wide credit spreads, before it ultimately takes place, i.e. the event is typically anticipated by market participants.

2 The models considered

In view of the classical trade-off between realism and tractability mentioned in (i) above, it is obvious that the realism increases with the axioms from (A1) to (A3), but to find tractable models also becomes increasingly more difficult. We consider three specific models in the sequel, all of them satisfying the no arbitrage condition that the wealth process (1) is a $\mathbb{Q}$-local martingale, but they differ with regards to axioms (A1) to (A3).

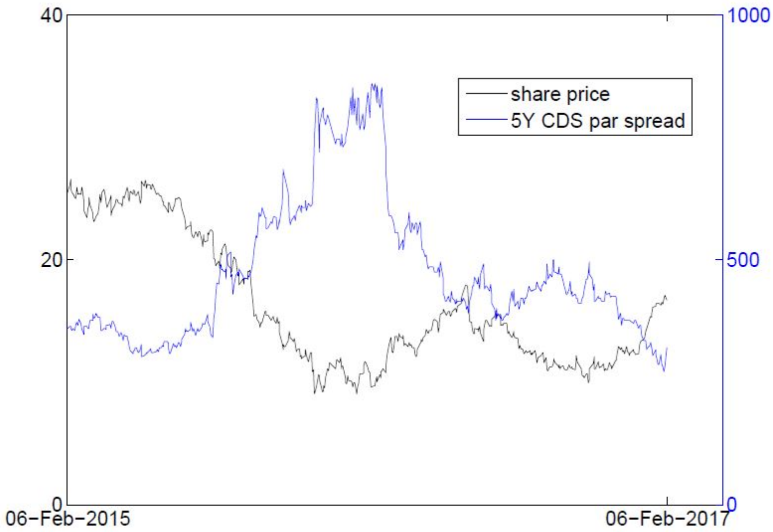

Fig. 1: Share price and 5Y CDS par spread of NRG Energy Inc., a US company, from 6 February 2015 to 6 February 2017. A strong anti-correlation is obvious, e.g. Spearman's Rho between both time series is -71%, justifying a model assuming that one time series is a strictly decreasing function of the other.

2.1 Simplest credit-equity model

The model satisfies (A1) but not (A2). The bankruptcy time τ is some random variable with survival function given by $\mathbb{Q}(\tau > t) = \exp\big(-\int_0^t \lambda(s)\,\mathrm{d}s\big)$ for a non-negative and integrable function $\lambda(.)$, called *default intensity*[2]. The share price is defined as

$$S_t := S_0\, e^{\int_0^t \lambda(s)+r(s)-\delta(s)\,\mathrm{d}s}\, e^{-\frac{1}{2}\sigma^2\,t+\sigma\,W_t}\, 1_{\{\tau>t\}},\quad t\geq 0,$$

for a volatility parameter $\sigma > 0$, and $W = \{W_t\}_{t\geq 0}$ denotes a standard $\mathbb{Q}$-Brownian motion that is independent of τ. Within this model, the expectation value in (2) boils down to an integral with respect to a lognormal density, and the survival probabilities in (3) are by definition given in terms of the default intensity $\lambda(.)$, and fully independent of σ.

2.2 JDCEV model

The model satisfies (A2) but not (A3). The specification we consider is a three-parametric special case of the model introduced in Carr, Linetsky (2006). With parameters $\lambda_0, \sigma_0 > 0$, and $\beta < 0$ it is based on a diffusion process $Z = \{Z_t\}_{t\geq 0}$ given by the SDE

$$\frac{\mathrm{d}Z_t}{Z_t} = \big(r(t)-\delta(t)+\lambda(Z_t)\big)\,\mathrm{d}t + \sigma(Z_t)\,\mathrm{d}W_t,\; Z_0 = S_0,$$
$$\lambda(z) := \lambda_0\left(\frac{z}{S_0}\right)^{2\,\beta},\quad \sigma(z) := \sigma_0\left(\frac{z}{S_0}\right)^{\beta},$$

where $W = \{W_t\}_{t\geq 0}$ is a standard $\mathbb{Q}$-Brownian motion. With ϵ denoting an exponential random variable with unit mean, independent of W, the bankruptcy time and the share price are

[2] If $\lambda(.) \equiv \lambda$ is identically constant, τ has an exponential distribution.

defined as

$$\tau := \inf\big\{t > 0 \,:\, \int_0^t \lambda(Z_s)\,\mathrm{d}s > \epsilon\big\}, \tag{4}$$
$$S_t := Z_t\, 1_{\{\tau > t\}}, \quad t \geq 0.$$

For $\beta \searrow 0$, the model converges to the simplest credit-equity model with constant default intensity $\lambda(.) \equiv \lambda_0$. From this point of view it may be seen as a proper generalization of the latter. The JDCEV model has the striking property that there exist closed formulas for the values of European equity derivatives given in (2) and for survival probabilities as required in (3), see Carr, Linetsky (2006). However, all three model parameters $\lambda_0, \sigma_0, \beta$ enter both formulas, which is a difference to the simplest credit-equity model (where the parameter σ only affected equity derivatives but not credit derivatives).

2.3 JDCEV+ model

The model satisfies (A3) and includes the JDCEV model as a special case. The intuition behind the model is to enhance the JDCEV model in such a way that sudden, unexpected defaults cannot happen. In order to describe its idea intuitively, we put ourselves in the setup of the JDCEV model and consider one particular scenario ω in which default happens before maturity T, i.e. $\tau(\omega) \leq T$. The share price path $t \mapsto S_t(\omega)$ drops to zero at $\tau(\omega)$, but this default can happen in two qualitatively different ways: Either the observable share price decreases dramatically already on $[0, \tau)$, which leads to a huge value $\int_0^{\tau(\omega)} \lambda(S_s(\omega))\,\mathrm{d}s > \epsilon(\omega)$ triggering default according to the definition of τ in (4). Or this is not the case and the default happens because the unobservable variable $\epsilon(\omega)$ turns out unexpectedly small. While the first scenario is in accordance with axiom (A3), the second is not. Thus, we seek to enhance the JDCEV model in such a way as to eliminate scenarios of the second kind.

First, we enhance the JDCEV model by a parameter $\eta \in (0,1)$ as follows. On a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ we consider two unit exponential random variables $\epsilon, \tilde{\epsilon}$, and two standard Brownian motions $W, \tilde{W}$, all four objects stochastically independent. The diffusion process Z is replaced by a process $\tilde{Z}$ according to the SDE

$$\frac{\mathrm{d}\tilde{Z}_t}{\tilde{Z}_t} = \big(r(t) - \delta(t) + \eta\,\tilde{\lambda}(\tilde{Z}_t)\big)\,\mathrm{d}t + \tilde{\sigma}(\tilde{Z}_t)\,\mathrm{d}\tilde{W}_t,\ \tilde{Z}_0 = S_0,$$
$$\tilde{\lambda}(z) := \tilde{\lambda}_0\left(\frac{z}{S_0}\right)^{2\tilde{\beta}}, \quad \tilde{\sigma}(z) := \tilde{\sigma}_0\left(\frac{z}{S_0}\right)^{\tilde{\beta}}.$$

Let $\tilde{\tau}$ be defined exactly as τ is defined in the JDCEV model, only replacing Z by $\tilde{Z}$, that is

$$\tilde{\tau} := \inf\big\{t > 0 \,:\, \int_0^t \tilde{\lambda}(\tilde{Z}_s)\,\mathrm{d}s > \tilde{\epsilon}\big\},$$

and consider the process

$$\tilde{S}_t := \begin{cases} \tilde{Z}_t & \text{, if } t < \tilde{\tau} \\ (1-\eta)\,\tilde{Z}_{\tilde{\tau}} & \text{, if } t \geq \tilde{\tau} \end{cases}, \quad t \geq 0.$$

The sole difference between the JDCEV model (τ, S) and the model $(\tilde{\tau}, \tilde{S})$ is that the share price $\tilde{S}$ does not jump down to zero at $\tilde{\tau}$, like S does at τ, but instead it looses the percentage η of its value at $\tilde{\tau}$ and then remains frozen at that value until eternity. For $\eta \nearrow 1$ the model converges to the JDCEV model. It can be shown that the wealth process (1) with respect to the share price $\tilde{S}$ is a local $\mathbb{Q}$-martingale with respect to the filtration

$$\tilde{\mathcal{F}}_t := \sigma\big(\tilde{Z}_s,\, 1_{\{\tilde{\tau}>s\}}\,:\, s \leq t\big), \quad t \geq 0,$$

so this alternative model, in which the stock does not jump all the way down to zero, is an arbitrage free pricing model as well. However, we do not interpret the time point $\tilde{\tau}$ as bankruptcy time, but as a time point at which distress information regarding to the company is made public in the marketplace, and hence causes a massive loss. The company is assumed to live on after $\tilde{\tau}$ in another distressed regime, which is again modeled by a second JDCEV model with much extremer parameters. To be precise, we consider a JDCEV diffusion with parameters and start value being randomly given as functions of $\tilde{S}_{\tilde{\tau}}$, given by the SDE

$$\frac{\mathrm{d}Z_t}{Z_t} = \big(r(t) - \delta(t) + \lambda(Z_t)\big)\,\mathrm{d}t + \sigma(Z_t)\,\mathrm{d}W_t,\; Z_0 = \tilde{S}_{\tilde{\tau}},$$
$$\lambda(z) := \lambda_0(\tilde{S}_{\tilde{\tau}})\left(\frac{z}{\tilde{S}_{\tilde{\tau}}}\right)^{2\,\beta(\tilde{S}_{\tilde{\tau}})}, \quad \sigma(z) := \sigma_0(\tilde{S}_{\tilde{\tau}})\left(\frac{z}{\tilde{S}_{\tilde{\tau}}}\right)^{\beta(\tilde{S}_{\tilde{\tau}})},$$

where the parameters $\big(\lambda_0(.), \sigma_0(.), \beta(.)\big)$ are functions of the share price $\tilde{S}_{\tilde{\tau}}$ at $\tilde{\tau}$. Finally, the bankruptcy time and share price are defined as

$$\tau := \inf\Big\{t > \tilde{\tau}\,:\, \int_{\tilde{\tau}}^{t} \lambda(Z_{s-\tilde{\tau}})\,\mathrm{d}s > \epsilon\Big\},$$
$$S_t := \begin{cases} \tilde{S}_t & \text{, if } t \leq \tilde{\tau} \\ Z_{t-\tilde{\tau}}\, 1_{\{\tau>t\}} & \text{, else} \end{cases}, \quad t \geq 0.$$

The parameters $\big(\lambda_0(.), \sigma_0(.), \beta(.)\big)$ are chosen “extreme” in the sense that a soon default after $\tilde{\tau}$ is highly likely. For our case study below, we choose

$$\lambda_0(\tilde{S}_{\tilde{\tau}}) = \tilde{\lambda}_0\left(1 + 6\left(1 - 2^{-\frac{\tilde{S}_{\tilde{\tau}}}{S_0}}\right)\right),$$
$$\sigma_0(\tilde{S}_{\tilde{\tau}}) = \tilde{\sigma}_0, \quad \beta(\tilde{S}_{\tilde{\tau}}) = \tilde{\beta}.$$

While the parameters $\tilde{\beta}$ and $\tilde{\sigma}_0$ remain unchanged after $\tilde{\tau}$, the idea of the definition of $\lambda_0(\tilde{S}_{\tilde{\tau}})$ is to increase it more if $\tilde{S}_{\tilde{\tau}}$ is higher. If $\tilde{S}_{\tilde{\tau}}$ is already close to zero, this parameter remains almost unchanged. Thus, this definition does not change paths of the first JDCEV model that are already doomed for default, but paths of the first JDCEV model in which $\tilde{\tau}$ comes by complete surprise are turned into paths of a new JDCEV model with very high instantaneous default probability to likely trigger a default shortly after $\tilde{\tau}$. Figure 2 visualizes two paths of this model.
We define the market filtration $(\mathcal{F}_t)_{t\geq 0}$ as

$$\mathcal{F}_t := \begin{cases} \tilde{\mathcal{F}}_t & \text{, if } t \leq \tilde{\tau} \\ \tilde{\mathcal{F}}_{\tilde{\tau}} \vee \sigma\big(Z_{s-\tilde{\tau}},\, 1_{\{\tau>s\}}\,:\, \tau < s \leq t\big) & \text{, else} \end{cases}, \quad t \geq 0,$$

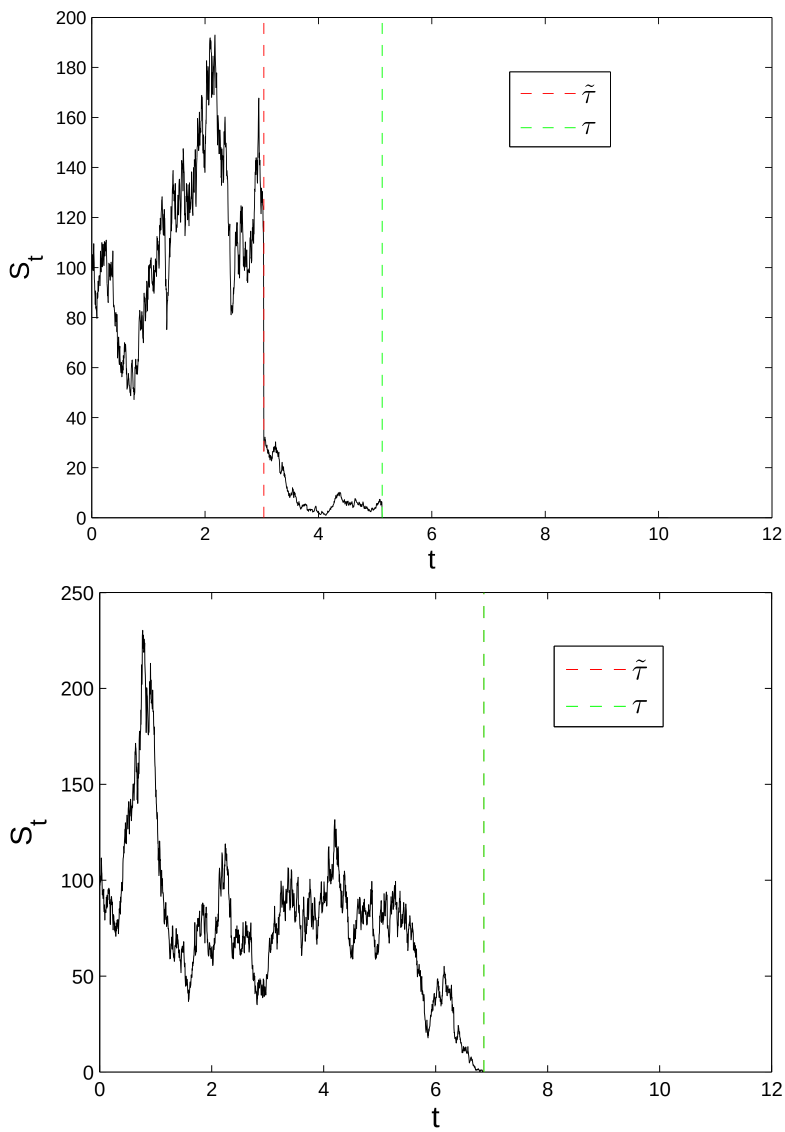

Fig. 2: Two paths of the JDCEV+ model with parameters $S_0 = 100$, $\tilde{\lambda}_0 = 0.05$, $\tilde{\beta} = -0.4$, $\tilde{\sigma}_0 = 0.7$, $\eta = 0.75$. The upper plot shows a path in which $\tilde{\tau}$ comes by complete surprise and the share price is doomed for default after $\tilde{\tau}$. The lower plot shows a path in which $\tilde{\tau}$ is already anticipated by the market, and hence almost coincides with τ.

and note that the wealth process (1) is a $\mathbb{Q}$-local martingale with respect to $(\mathcal{F}_t)$. This follows from the fact that two JDCEV-martingales are pasted together at $\tilde{\tau}$ smoothly, i.e. the start value of the second martingale coincides with the end value of the first martingale. The time point $\tilde{\tau}$ is best thought of as a change point at which the parameters and the absolute value of the JDCEV share price change dramatically. After $\tilde{\tau}$ the model is a regular JDCEV model, which is started at $\tilde{\tau}$ with parameters and start value depending on the first JDCEV model's final value $\tilde{S}_{\tilde{\tau}}$.

3 A case study

Concerning the ability to capture realistic features, the JDCEV+ model is the most appropriate one among the three introduced models, since it satisfies all axioms (A1), (A2), and (A3). Unfortu-

nately, closed pricing formulas for credit- and equity-derivatives are challenging to evaluate numerically due to the mathematical complexity of the model. However, it is possible to evaluate derivative prices by means of a Monte Carlo simulation. Quickly explained, this means that a huge number N of independent simulations of the model $(\{S_t\}_{t\in[0,T]}, \tau)$ are generated in the computer. Each single simulation contributes a possible realization for the derivative prices in concern. Ultimately, estimates of the derivative prices under concern are given by the arithmetic average over all simulated prices. By the law of large numbers, these estimates converge to the true derivative prices as N tends to infinity, which justifies to use them as price evaluations for large N. In the following case study we set $N = 100,000$.
In the sequel, we consider four observed market CDS curves with differing qualitative nature, which capture different possible regimes a company can be in:

(a) **IG curve**: an upward sloping CDS curve with 1Y CDS par spread quite tight at around 20 bps, and 10Y CDS par spread at around 180 bps, which looks quite typical for an average investment grade company.

(b) **Moderate spread levels**: an upward sloping CDS curve with 1Y CDS par spread at around 160 bps, and 10Y CDS par spread at around 475 bps.

(c) **HY curve**: an upward sloping CDS curve with 1Y CDS par spread already quite wide at around 500 bps, and 10Y CDS par spread at around 830 bps, which looks quite typical for an average high yield company.

(d) **Distressed curve**: a CDS curve with 1Y CDS par spread already at around 890 bps, 3Y CDS par spread at 1100 bps, and then decreasing with a 10Y CDS par spread still at around 950 bps, which looks quite typical for a highly distressed company.

For each of the cases (a) to (d) we find model parameters of the JDCEV+ model explaining the desired CDS curve shapes, the prices being computed by Monte Carlo simulation. The parameters of the JDCEV model and of the simple credit-equity model are then calibrated to the same CDS curve. For the simple credit-equity model this calibration works perfectly by defining the default intensity function $\lambda(.)$ in a piecewise constant manner and bootstrapping the single pieces iteratively to the JDCEV+ CDS prices, as described, e.g., in O'Kane, Turnbull (2003). For the considered three-parametric JDCEV model, the calibration does not provide a perfect, but a satisfactorily good match to the JDCEV+ CDS prices.
Figures 3, 4, 5, and 6 visualize the results in cases (a), (b), (c), and (d), respectively. The top plots in all figures show the CDS curves implied by the JDCEV and the JDCEV+ model. These are almost identical in all figures, as desired, demonstrating the good fitting capacity of the JDCEV model to the CDS curve computed via Monte Carlo simulation in the JDCEV+ model. The CDS curve implied by the simple credit-equity model is not depicted,

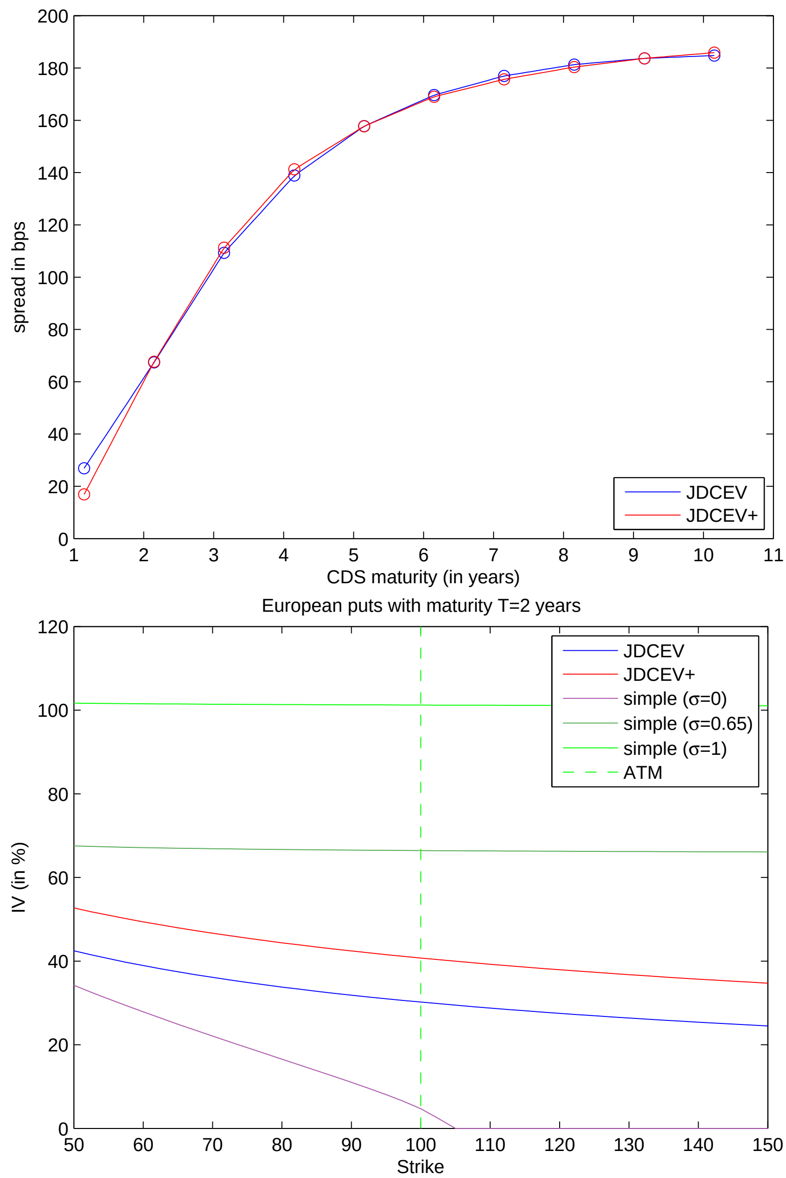

Fig. 3: Top: CDS curves implied by the JDCEV and the JDCEV+ model in case (a). Bottom: Implied volatility smiles implied from European put options with two years maturity in all considered models.

since it is clearly possible to match any CDS curve accurately by bootstrapping a piecewise constant default intensity. The bottom plots in all figures visualize the implied volatility smiles for European put options with two year maturity. These are depicted for JDCEV and JDCEV+ model, as well as for different simple credit-equity models, namely with three different choices for the parameter $\sigma \in \{0, 0.65, 1\}$. Recall that σ in the simple credit-equity model has no effect on the CDS curve, so can be chosen arbitrarily without changing the model-implied CDS curve, which is a major difference compared with the other two considered models.

It is observed that in those cases with upward sloping CDS curve, namely cases (a), (b), and (c), the implied volatility smile in the JDCEV model lies below the implied volatility smile in the

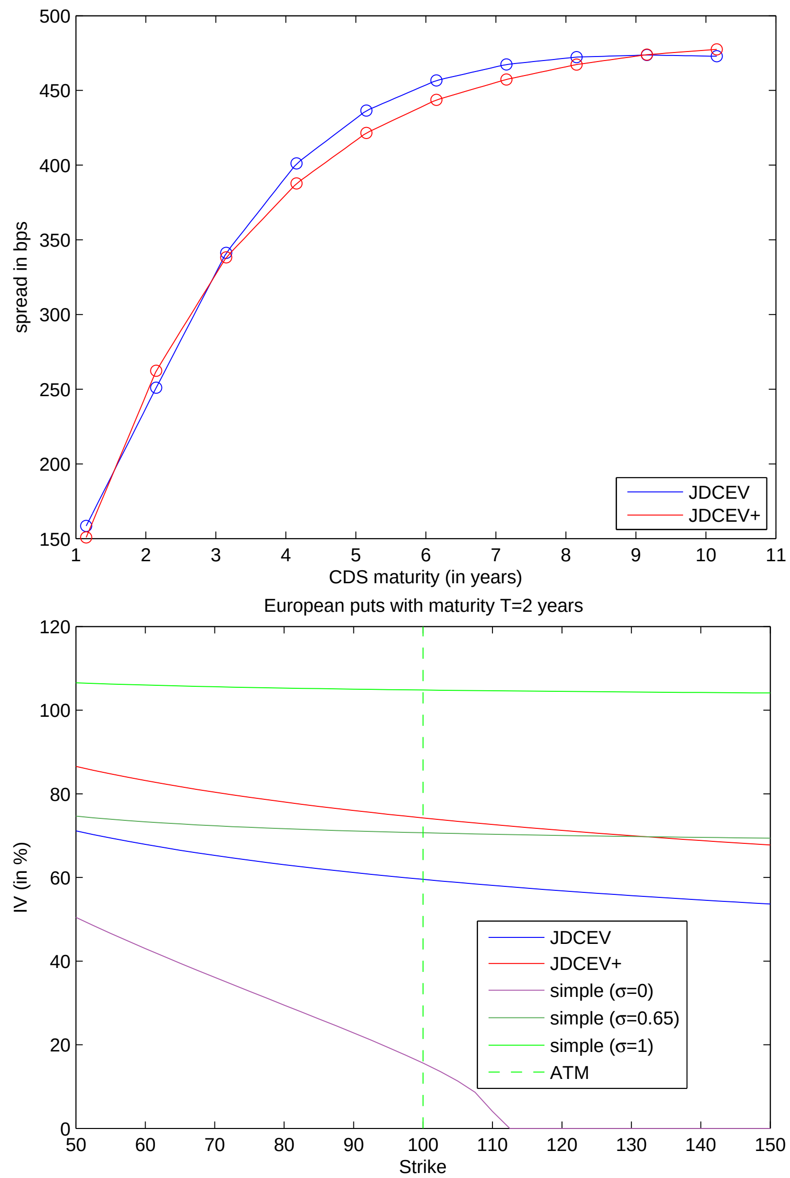

Fig. 4: Top: CDS curves implied by the JDCEV and the JDCEV+ model in case (b). Bottom: Implied volatility smiles implied from European put options with two years maturity in all considered models.

JDCEV+ model. Only in case (d) with inverse CDS curve the JDCEV model exhibits higher implied volatilities than the JDCEV+ model. In the IG curve case (a) the difference between the JDCEV+ and the JDCEV model in volatility points for the considered options is around 10, in case (b) around 15, in case (c) around 10 again, and in case (d) it becomes negative. Recalling the two qualitatively different types of paths in the JDCEV+ model from Figure 2, an explanation could be as follows. The JDCEV+ model features almost no paths in which default comes completely unanticipated (which is its major motivation). Consequently, in order to obtain the same CDS curve (i.e. default probabilities) in JDCEV and JDCEV+ model, the latter requires a higher level of spread- (hence stock-) volatility in order to compensate for that part of the CDS price accounting for "surprising defaults" in the JDCEV model. This higher stock volatility apparently carries over to higher option prices, i.e. the option prices seem to be more sensitive to "volatility" than to "pure sudden default risk" in normal situations (cases (a), (b) and (c)). When the situation is already very distressed (such as in case (d)), this phenomenon becomes reversed, i.e. option prices seem to be more sensitive to "pure sudden default risk" (present only in JDCEV model) than to "volatility" (stronger present in JDCEV+ model).

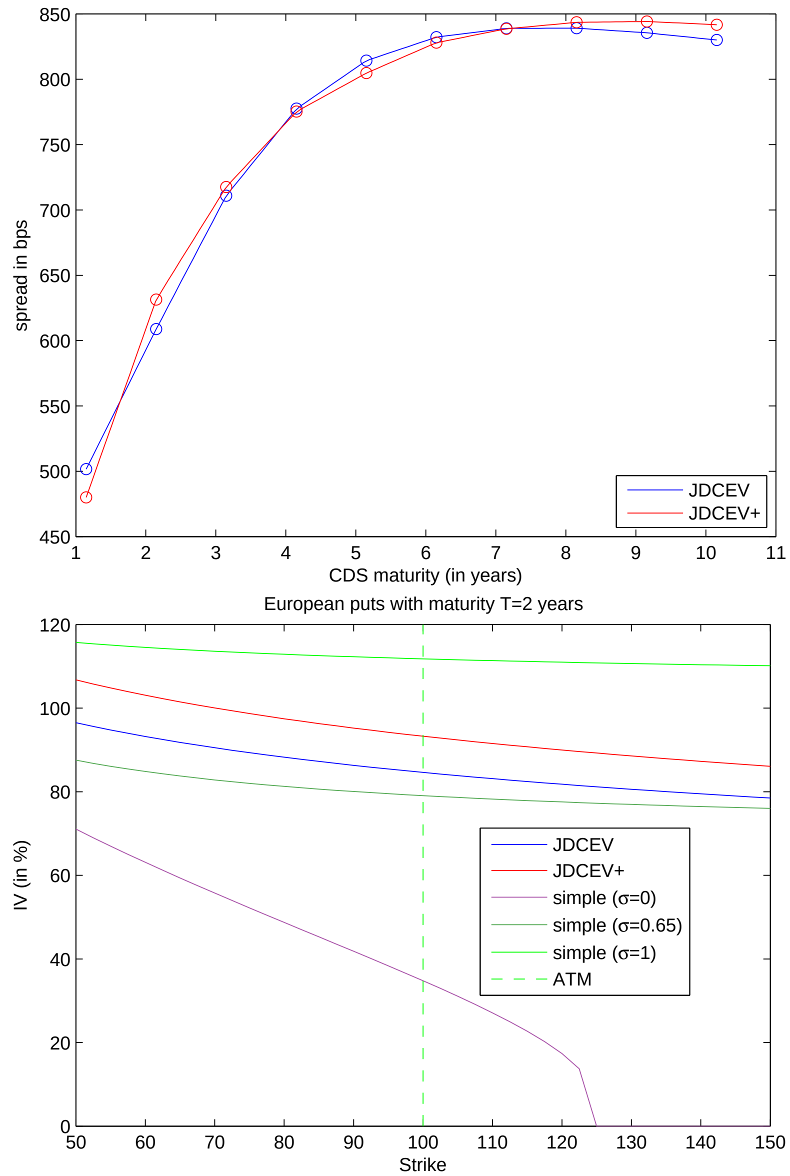

Fig. 5: Top: CDS curves implied by the JDCEV and the JDCEV+ model in case (c). Bottom: Implied volatility smiles implied from European put options with two years maturity in all considered models.

Having a look at the implied volatility smiles induced by the different simple credit-equity models, one realizes that the free parameter σ essentially controls the absolute level of the implied volatility smile. With increasing σ the implied volatilities increase. As σ tends to zero, the implied volatility smile tends to a lower bound (the purple line), which is higher the wider the CDS curve is. This is intuitive, since wider CDS par spreads imply higher default probabilities, inducing higher probabilities for the event $\{S_T = 0\}$ in accordance with axiom (A1). By the martingale property of the wealth process, higher probabilities for the event $\{S_T = 0\}$ must come along with higher probabilities for S_T being large as well, hence with a higher variance of S_T, explaining higher option prices.

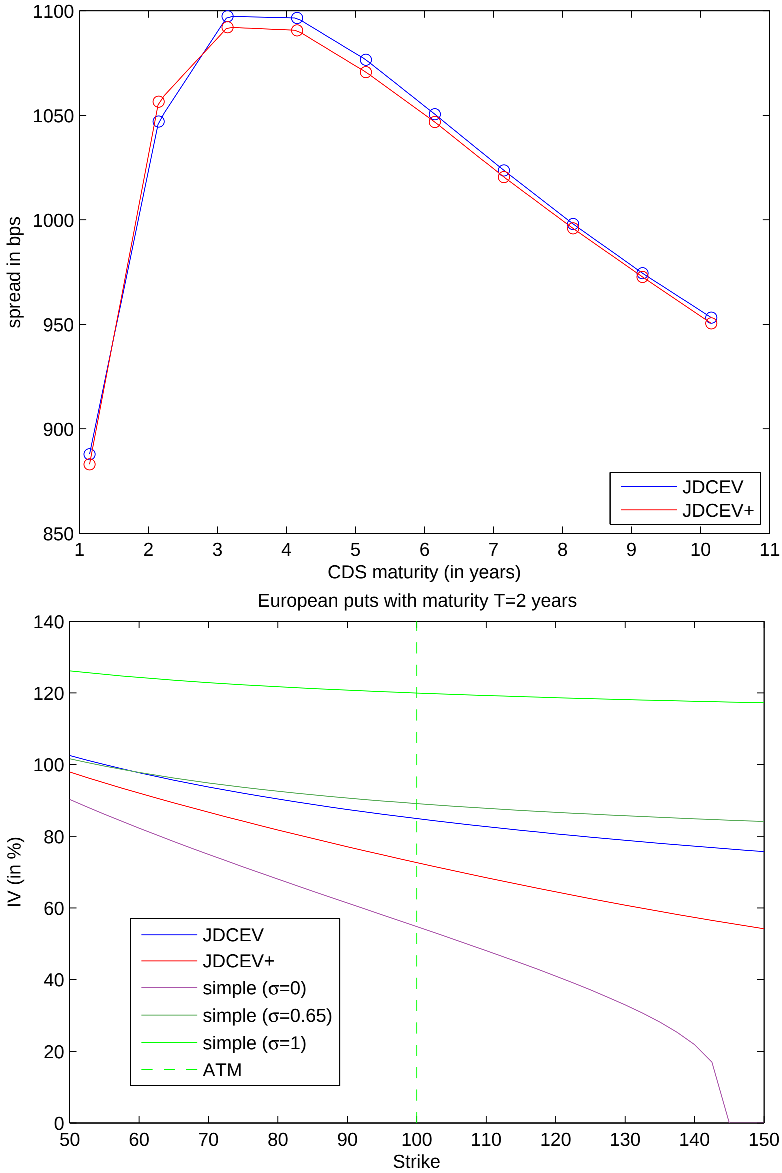

Fig. 6: Top: CDS curves implied by the JDCEV and the JDCEV+ model in case (d). Bottom: Implied volatility smiles implied from European put options with two years maturity in all considered models.

What is the conclusion of this case study? While both the JDCEV and the JDCEV+ model can be viewed as functions mapping an observed CDS curve uniquely to an implied volatility smile, the simple credit-equity model can not. The assumptions made in the simple credit-equity model to link credit- and equity-components are simply too weak to use it as a tool to infer information about one component from the other. At least the lower bound on the implied volatility smile in the case $\sigma = 0$ can be a useful information to be read off from the simple credit-equity model. A decision between JDCEV and JDCEV+ model is clearly more art than science. Whereas the JDCEV+ model might be one's preferred choice as it satisfies the most realistic axiom (A3), usability clearly favors the JDCEV model.

4 Conclusion

It was demonstrated that it is necessary to incorporate at least a reciprocal relationship between credit spread and equity into a credit-equity model in order to make it useful for predictions of credit derivatives prices from equity derivatives prices, or vice versa. Furthermore, when enhancing the so-called JDCEV model so as to eliminate paths with a sudden, unanticipated default, CDS-implied predicted implied volatilities in the model were observed to to be larger than in the usual JDCEV model in the usual case of an upward sloping CDS curve. In case of an inverse CDS curve, the opposite has been observed.

References

P. Carr, V. Linetsky, A jump to default extended CEV model: an application of Bessel processes, *Finance and Stochastics* **10:3** (2006) pp. 303–330.

F. Delbaen, W. Schachermayer, A General Version of the Fundamental Theorem of Asset Pricing, *Mathematische Annalen* **300**(1) (1994), pp. 463–520.

F. Delbaen, W. Schachermayer, The Fundamental Theorem of Asset Pricing for Unbounded Stochastic Processes, *Mathematische Annalen* **312** (1998), pp. 215–250.

D. O'Kane, S. Turnbull, valuation of credit default swaps, *Fixed Income Quantitative Research Lehman Brothers* (2003).