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The effect of the recovery rate when stressing par CDS spreads

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Abstract One popular procedure in risk management is to conduct stress tests, which should shed light on what happens in an extreme scenario. A particular stress scenario for credit portfolios is a credit spread widening. For CDS contracts an intuitive definition of such a scenario is to postulate that all par CDS spreads are blown out by a certain factor, leaving all other parameters unchanged. Even though two CDS contracts may have par CDS spreads of similar size, the present note explains why such a definition may affect the market value of one CDS more severely than the other when the two names have different recovery assumptions. More precisely, the upfront (i.e. market value) of the CDS with smaller recovery assumption is blown out further than the one of the CDS with bigger recovery assumption.

Par CDS spreads

A credit default swap (CDS) is a standardized insurance contract between two parties. The protection buyer pays a quarterly premium c (called the running coupon), and in return the protection seller compensates the protection buyer for potential losses due to a so-called credit event of a third party, the contractually specified reference entity. The credit event is specified in the CDS and can be a debt restructuring, a bankruptcy filing, the failure to pay a coupon etc., and the decision whether a credit event has occured or not is made by the ISDA Determinations Committee. In case a credit event has occured, an auction process is triggered in which all protection buyers are allowed to deliver eligible bonds, the outcome of which is a so-called recovery rate $R \in [0, 1]$. Consequently, the CDS protection seller has to pay to the protection buyer the CDS nominal times $(1 - R)$. Since the premium c is standardized (typically to 1% or 5%), but the insurance risk depends on the reference entity's creditworthiness, a CDS contract has a market value, which has to be paid by the protection buyer at inception of the contract, the so-called upfront payment. The upfront payment is typically quoted in percent of the CDS nominal, just as a bond price is quoted in percent of its nominal. Analogous to bonds, for a CDS protection seller it is more comfortable to think of the credit risk associated with a CDS contract in terms of a so-called "credit spread" that is interpreted as an annualized risk premium that can be earned for taking the credit risk associated with the reference entity. The so-called *par CDS spread* is the market standard quantity for this purpose. Given a recovery rate assumption R, a term structure of probabilities for the occurence of a credit event with respect to the reference entity, and a discounting term structure, it is defined as the unique insurance premium which the insurance buyer must pay so that the expected, discounted premium payments of insurance buyer equal the expected, discounted default compensation payments to be made by the insurasnce seller. In other words, if the premium c was replaced by the par CDS spread, the CDS market value, i.e. its upfront, equaled zero. When CDS began to be traded in the marketplace, the premium c was non-standardized and equaled precisely the par CDS spread, so that no initial cash exchange between insurance buyer and payer needed to take place. Even though this has changed, for historical reasons the typical market quotation of CDS is still in terms of par CDS spreads, instead of the actual market values, i.e. upfronts. This is a difference to bond markets, where it is rather unusual to quote bonds in terms of their yield or their Z-spread. Since the computation of a par CDS spread from a CDS market value requires recovery rate, discounting term structure, and credit event occurence probability assumptions, there exist market standard assumptions, constituted by ISDA. Depending on the CDS standard (there exist several standards in the market) and the seniority of the CDS contract (there exist senior and subordinated CDS), there is a different standard ISDA recovery assumption[1]. The standard discounting curve to be used is retrieved in a prescribed way from a standardized battery of swap rates in the CDS currency. The involved probabilities for the occurence of a credit event are computed from an exponentially distributed random variable whose exponential rate parameter is matched to the market value of the CDS. For the sake of simplified notation, in the following sections we present mathematical formulas based on even simpler discounting assumptions, on the assumption of continuous instead of discrete premium payments, and neglecting accrued coupon payments due in case of a credit event. These simplifications have no influence on the general message of the present note.

The credit triangle

Consider a CDS contract with maturity T and running coupon c. We assume that a credit event with respect to this CDS happens at the random time point τ, which we assume to be exponentially distributed with intensity parameter λ, and we assume that the recovery rate in case of a credit event equals the fixed amount $R \in [0,1]$. For the sake of simplified notation we assume a flat risk-free interest rate r for discounting cash flows, and that CDS coupon payments are made continuously. Under these assumptions, the CDS upfront (i.e. its market value from point of view of the protection buyer) is given by

$$\begin{aligned}\text{upf} &= (1-R)\,\mathbb{E}\Big[e^{-r\,\tau}\,1_{\{\tau\leq T\}}\Big] - c\int_0^T e^{-r\,t}\,\mathbb{P}(\tau>t)\,\mathrm{d}t\\ &= \frac{1-e^{-(r+\lambda)\,T}}{r+\lambda}\,\big(\lambda\,(1-R)-c\big).\end{aligned}$$

The *par CDS spread* s is defined as the unique running coupon c which makes the upfront vanish. The previous equation shows that it is given under our assumptions as

$$s = \lambda\,(1-R). \tag{1}$$

[1]See, e.g., http://www.cdsmodel.com/cdsmodel/fee-computations.html.

Equation (1) is called the *credit triangle*, because it constitutes a simple-to-remember relationship between three fundamental CDS components: the par CDS spread, the default intensity (which parameterizes default probabilities), and the recovery rate. Rearranging the credit triangle for λ and inserting into the upfront formula, we obtain the upfront as a function of the par CDS spread s and the recovery rate R as

$$\text{upf} = \text{upf}(s, R) = \frac{1 - e^{-(r+\frac{s}{1-R})T}}{r + \frac{s}{1-R}} \big(s - c\big).$$

The upfront is not only an increasing function in the par CDS spread s, but also a decreasing function in the recovery rate R, provided that $s > c$. This has important consequences for stress testing, as the following example demonstrates.

Stressing par CDS spreads

In a stress test, one typical scenario that is computed is typically that several risk factors in one's pricing library are stressed. For credit instruments this typically means that credit spreads are "blown out" (i.e. dramatically increased) in order to see how this affects the overall portfolio value. For CDS contracts, a typical asumption is to blow out the par CDS spread, since the latter represents the best known credit spread figure in the marketplace. Consider now two CDS contracts with similar covenants, i.e. similar maturity, running coupon, currency etc.. For the sake of simplified presentation we assume that both CDS have even the same maturity T, the same running spread c, and the same interest rate parameter r is used for discounting cash flows. Furthermore, we assume that the observed upfronts of both contracts are the same. However, for both contracts we make different recovery assumptions. For the first contract we assume the recovery rate R_1 and for the second contract $R_2 > R_1$. This implies that the par CDS spreads of the two contracts, denoted by s_1 and s_2, are potentially different. To be precise, we have upf$(s_1, R_1) =$ upf(s_2, R_2). During the stress test, we blow both par spreads s_1 and s_2 out by the same factor $x > 1$. The question we like to address is: which upfront increases stronger as a consequence of the stress test, i.e. do we have upf$(x\, s_1, R_1) >$ upf$(x\, s_2, R_2)$ or vice versa?
Figure 1 visualizes the function upf(s, R). Each point on the intersection line with the green area in the plot indicates a pair (s, R) of par CDS spread and recovery rate which yields an upfront of 20%. The two CDS contracts under consideration might be identified with two distinct points (s_1, R_1) and (s_2, R_2) on that line. Two antidromic effects can now be observed:

(a) On the one hand, it is observed that the derivative $\frac{\partial}{\partial s}upf(s, R)$ of the upfront in the spread-direction is higher, the lower the recovery rate R is. This suggests that the stress imposed on the first CDS with lower recovery R_1 leads to a more significant increase in the respective upfront than the increase induced by the stress imposed on the second CDS, i.e. we intuitively expect upf$(s_1 + \Delta, R_1) >$ upf$(s_2 + \Delta, R_2)$ for $\Delta > 0$.

(b) On the other hand, we have that $\Delta_1 := x\, s_1 - s_1$ is smaller or equal compared with $\Delta_2 := x\, s_2 - s_2$, simply because $s_1 \leq$

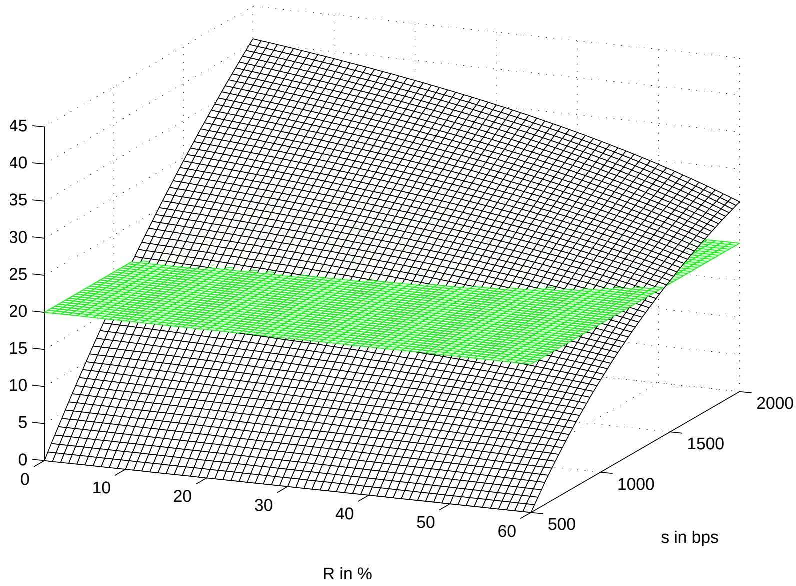

Fig. 1: Visualization of the function upf(s, R) in $\%$ with parameters $c = 500$ bps, $T = 4$, and $r = 0.01$. The intersection with the green area visualizes the spread and upfront levels for which the upfront equals 20%.

s_2 (which is a consequence of upf(s, R) being a decreasing function in R, provided $s > c$).

Consequently, it is a priori unclear whether we have upf$(x\, s_1, R_1) >$ upf$(x\, s_2, R_2)$ or vice versa. However, Figure 1 indicates that the effect (b) is secondary compared to effect (a), so that we have indeed that upf$(x\, s_1, R_1) >$ upf$(x\, s_2, R_2)$. In one particular case this becomes clearly visible in Figure 1, namely if we assume that the observed upfronts of both contracts are zero and $x = 4$. In this case $s_1 = s_2 = 500$ bps and $x\, s_1 = x\, s_2 = 2000$ bps, so that $\Delta_1 = \Delta_2$ and effect (b) disappears completely. The blown out upfront in dependence on the recovery rate, i.e. the function $R \mapsto$ upf$(2000$ bps$, R)$, is obviously decreasing. In other words, the upfront upf$(x\, s_1, R_1)$ of the CDS with lower recovery rate assumption R_1 is blown out further than the upfront upf$(x\, s_2, R_2)$ of the second CDS with higher recovery rate R_2.