Abstract
For standard CDS the recovery rate and thus the protection payment in case of a default event depends on the reference obligation's price calculated in the auction process after a default event, which implies an uncertainty in the protection payment. In some cases, e.g., when a credit event seems to be quite probable, an investor aims to eliminate this uncertainty. Recovery Swaps provide a way to do so. This article gives a short introduction to recovery products and their mechanisms and highlights the calculation of the invested capital for Recovery Swaps in practice.

1 What is a Recovery Swap?
Talking about a default (event) in the following, we always refer to a credit event occurring in the underlying reference obligation prior to maturity of the contract.

The three mainly traded recovery products are:

- Fixed Recovery CDS,
- Recovery Locks,
- Recovery Swaps.

A Fixed Recovery CDS (or Digital Default CDS) is a standard CDS where the (contractual) recovery rate $R_{fix}$ is fixed when entering into the contract. Thus, the Fixed Recovery Buyer pays a quarterly premium to the Fixed Recovery Seller and in case of a default event he receives $(1 - R_{fix}) \times N$, where $N$ is the contract nominal. As well as for standard CDS the terms buyer and seller refer to the bought and sold protection, respectively. Figure 1 illustrates the cashflows for a Fixed Recovery CDS, where the continuous line represents the cashflows before and the dotted line the cashflow after default event (if any).

A Recovery Lock is a contract where both parties, called Recovery Lock Receiver and Recovery Lock Payer, agree to exchange the initially fixed recovery rate $R_{fix}$ and actual recovery rate $R_{actual}$ in case of a default event. Here, the terms payer and receiver refer to the realised recovery rate $R_{actual}$. For this kind of contract, there are neither running coupons nor upfront payments. This implies that there is no cashflow unless a credit event occurs, see Figure 2. Hence, the initially fixed recovery rate equals the market implied recovery rate when entering into the contract.

The third instrument we are looking at in this article are Recovery Swaps which are in fact a combination of a short (long) standard CDS and long (short) Fixed Recovery CDS. More detailed, the
Recovery Swap payer sells a Fixed Recovery CDS and therefore receives the running coupon and has to pay \((1 - R_{\text{fix}}) \times N\) in case a default event occurs. Simultaneously, he is buyer of a standard CDS and pays running coupons and receives \((1 - R_{\text{actual}}) \times N\) in the default case. Assuming \(R_{\text{fix}} = R_{\text{actual}}\) when entering into the contract and as coupon and upfront payments obviously net out, the Recovery Swap's payoff is the same as for a Recovery Lock, see Figure 3.

At a first glance, it seems that Recovery Locks and Recovery Swaps are exactly the same instruments only having different names. This is true when looking only at the cash flows, but the main difference between Recovery Locks and Recovery Swaps is that for the Recovery Swap the two parts are physically two separate contracts and therefore, e.g., can be unwound separately. Although a Recovery Swap is the same as a Recovery Lock based on a pure cashflow perspective, it is very important in practice, however, to determine exactly which instrument is traded in
regards to settlement and regulatory demands. One says that the buyer of a standard CDS is short recovery as he receives a decreasing amount \((1 - R_{\text{actual}}) \times N\) for an increasing recovery rate. Likewise, a receiver of a Recovery Lock or Recovery Swap is long recovery.

A practical example for the application of Recovery Swaps is described in the following scenario. Assume having a short standard CDS in your portfolio, where the issuer of the reference obligation is likely to default in the near future. A default would imply a cashflow of \((1 - R_{\text{actual}})\) per unit notional to the protection buyer. Selling a Recovery Swap will eliminate the uncertainty of this amount (which has to be paid with high probability) as the cashflow (outflow) per unit notional in case of a default will be

\[
(1 - R_{\text{actual}}) \times N + (R_{\text{actual}} - R_{\text{fix}}) \times N = (1 - R_{\text{fix}}) \times N.
\]

Hence, for equal nominals of the standard CDS and Recovery Swap, recovery rate risk is fully hedged.

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2 Invested Capital To measure the performance of some portfolio it is common to use the carry measurement which is an *annualized, expected return on investment* defined by

\[
\mu_{1y} = \frac{\text{expected income (within next year)}}{\text{invested capital}}.
\]

The expected income for a short CDS might be defined as the sum of the expected discounted coupons within next year plus market value change, where an unchanged credit curve is assumed. As a definition for the invested capital we suggest that this is the capital that is potentially lost in the worst case. For long credit investments, the worst case is typically a default event and, thus, one immediately loses the market value on the one hand. The capital exceeding the market value which is lost upon an immediate default and required in order to perfectly compensate
the effect of an immediate default event is called capital at stake. Hence, the invested capital can be split as follows:

\[
\text{invested capital} = \text{market value} + \text{capital at stake}.
\]

This implies that for example the capital at stake for a bond is zero because a bond may at most lose its market value. However, the market value of a short standard CDS position might be negative, while there is of course some capital at stake because one faces the possibility of a default compensation payment.

In order to determine the capital at stake and finally the invested capital of Recovery Swaps it is helpful to start with short standard CDS. Being short a standard CDS leads to a payment of \((1 - R_{\text{actual}}) \times N\) if a default event occurs. The worst outcome case for the protection seller is a recovery rate of zero \((R_{\text{actual}} = 0)\). Hence, the capital at stake equals the nominal:

\[
\text{capital at stake} = N,
\]

which is obviously the most conservative assumption.

Now, suppose we have a portfolio consisting of only one short Recovery Swap, which implies an immediate outflow of \((R_{\text{actual}} - R_{\text{fix}}) \times N\) in case of a default event. Keeping the most conservative view, we get (for \(R_{\text{actual}} = 1\))

\[
\text{capital at stake} = (1 - R_{\text{fix}}) \times N.
\]

Assume now having a portfolio consisting of a short standard CDS and a short Recovery Swap, both having the same nominals \(N\). Notice, that treating the two instruments separately could cause mistakes in the calculation in practice as for keeping the most conservative view in one case \(R_{\text{actual}} = 1\) and in the other case \(R_{\text{actual}} = 0\) has to be applied. Rather, we have in this case

\[
\text{capital at stake} = (1 - R_{\text{actual}}) \times N + (R_{\text{actual}} - R_{\text{fix}}) \times N
= (1 - R_{\text{fix}}) \times N.
\]

The following table gives a short overview for the calculations of the invested capital for the above discussed instruments where \(MV\) denotes the corresponding market values of one unit nominal.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Invested Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>short standard CDS</td>
<td>((1 + MV_{CDS}) \times N)</td>
</tr>
<tr>
<td>short Recovery Swap</td>
<td>((1 - R_{\text{fix}} + MV_{RS}) \times N)</td>
</tr>
<tr>
<td>short CDS &amp; short Recovery Swap</td>
<td>((1 - R_{\text{fix}} + MV_{Portfolio}) \times N)</td>
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</tbody>
</table>

To achieve the correct calculation of the invested capital (including capital at stake) as shown above in practice for a portfolio containing \(n\) short standard CDS and \(m\) short Recovery Swaps we propose the following algorithm which takes into account the aforementioned netting effects appropriately:

1. Calculate the invested capital for all short standard CDS and sum up their nominals simultaneously.

\[
\text{invested capital} = \sum_{i=1}^{n} (1 + MV_{CDS_i} \times N_{CDS_i}),
\]
and 

\[ N_{CDS} := \sum_{i=1}^{n} N_{CDS_i}. \]

2. Choose the first short Recovery Swap \((j = 1)\).

3. Compare the currently chosen Recovery Swap’s nominal \(N_{RS_j}\) to \(N_{CDS}\): If \(N_{RS_j} \leq N_{CDS}\) go to 4., else go to 5.

4. Capital at stake netting can be applied, thus the invested capital becomes

\[
\text{invested capital} = \text{invested capital} + (MV_{RS_j} - R_{fix}) \times N_{RS_j}.
\]

Reduce the overall CDS nominal \(N_{CDS}\) by \(N_{RS_j}\), i.e. \(N_{CDS} = N_{CDS} - N_{RS_j}\).

If there is at least one remaining Recovery Swap (i.e. if \(j < m\)), choose the next one \((j = j + 1)\) and go to 3.

5. Capital at stake netting can not be applied, thus the invested capital becomes

\[
\text{invested capital} = \text{invested capital} + (MV_{RS_j + 1} - R_{fix}) \times N_{RS_j}.
\]

If there is at least one remaining Recovery Swap (i.e. if \(j < m\)), choose the next one \((j = j + 1)\) and go to 3.

**3 Pricing**

A simplified way for pricing Recovery Swaps is to virtually split the contract into a standard CDS and a Fixed Recovery Swap as we did for introducing the instrument and price the two parts separately. For pricing the standard CDS there exists for example the ISDA model which requires (among others) a credit curve and an assumed recovery rate as input. The same model can also be used for pricing the Fixed Recovery Swap but using the predetermined, fixed- instead of the assumed-recovery rate. The present value of the Recovery Swap is then the difference of both parts where the direction depends on payer/receiver view. An improvement for pricing the Recovery Swap is using a recovery term structure instead of one assumed recovery for all maturities (flat recovery curve). Unfortunately, recovery term structures are usually not observed in the market.


**References**